Ministry of Education

Saskatchewan Curriculum

# Mathematics

# 2

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## Acknowledgements

The Ministry of Education wishes to acknowledge the professional contributions and advice of the provincial curriculum reference committee members:

Daryl Bangsund
Good Spirit School Division
LEADS

Dr. Murray Bremner
Department of Mathematics and Statistics
University of Saskatchewan

Linda Goulet
Associate Professor
First Nations University of Canada

Angie Harding
Regina Roman Catholic Separate School Division
Saskatchewan Teachers' Federation

Susan Jeske
Prairie Spirit School Division
Saskatchewan Teachers' Federation

Wendy Lang
St. Paul's Roman Catholic Separate School Division
Saskatchewan Teachers' Federation

George McHenry
Division Board Trustee
Saskatchewan School Boards Association

Dr. Shaun Murphy
College of Education
University of Saskatchewan

Dr. Kathy Nolan
Faculty of Education
University of Regina

Kathi Sandbeck
Sun West School Division
Saskatchewan Teachers' Federation

Doug Sthamann
Regina Public School Division
Saskatchewan Teachers' Federation

Rodney White
North East School Division
Saskatchewan Teachers' Federation

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# Introduction

As a required area of study, Mathematics is to be allocated 210 minutes per week for the entire school year at Grade 2. It is important that students receive the full amount of time allocated to their mathematical learning and that the learning be focused upon students attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

The outcomes in Grade 2 Mathematics build upon students' prior learnings and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These continuing learnings prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

*Outcomes describe the knowledge, skills, and understandings that students' are expected to attain by the end of a particular grade level.*

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to know and/or be able to do in order to achieve the learnings intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can and should be created by teachers to meet the needs and circumstances of their students and communities.

*Indicators are a representative list of the types of things a student should know or be able to do if they have attained the outcome.*

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Grade 2 Mathematics outcomes have been influenced by the renewal of the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K-9 Mathematics* outcomes (2006). Changes throughout all of the grades have been made for a number of reasons including:

- decreasing content in each grade to allow for more depth of understanding
- rearranging concepts to allow for greater depth of learning in one year and to align related mathematical concepts
- increasing the focus on numeracy (i.e., understanding numbers and their relationship to each other) beginning in Kindergarten
- introducing algebraic thinking earlier.

Also included in this curriculum is information regarding how Grade 2 Mathematics connects to the K-12 goals for mathematics. These goals define the purpose of mathematics education for Saskatchewan students.

*In Grade 2, students learn about equality and inequality, both of which are foundations of algebraic thinking.*

In addition, teachers will find discussions of the critical characteristics of mathematics education, assessment and evaluation of student learning in mathematics, inquiry in mathematics, questioning in mathematics, and connections between Grade 2 Mathematics and other Grade 2 areas of study within this curriculum.

Finally, the Glossary provides explanations of some of the mathematical terminology you will find in this curriculum.

## Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its various components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (August 2007) on the Ministry of Education website.

## Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 Mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

### Building Lifelong Learners

*Related to the following Goals of Education:*
- *Basic Skills*
- *Lifelong Learning*
- *Self Concept Development*
- *Positive Lifestyle*

*Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
*(NCTM, 2000, p. 20)*

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 Mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 Mathematics). It is crucial that the students discover the mathematics outlined in the curriculum rather than the teacher covering it.

## Building a Sense of Self and Community

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue are exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Mathematics also provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students will develop their personal and social identity, and learn healthy and positive ways of interacting and working together with others.

*Related to the following Goals of Education:*
- *Understanding & Relating to Others*
- *Self Concept Development*
- *Positive Lifestyle*
- *Spiritual Development*

*Many of the topics and problems in a mathematics classroom can be initiated by the children themselves. In a classroom focused on working mathematically, teachers and children work together as a community of learners; they explore ideas together and share what they find. It is very different to the traditional method of mathematics teaching, which begins with a demonstration by a teacher and continues with children practicing what has been demonstrated.*

*(Skinner, 1999, p. 7)*

## Building Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students such as trends in global warming, homelessness, technological health issues (oil spills, hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students can derive through mathematical analysis, they become better informed and have a greater respect for and understanding of differing opinions and possible options. With

*Related to the following Goals of Education:*
- *Understanding & Relating to Others*
- *Positive Lifestyle*
- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change*

these understandings, students can make better informed and more personalized decisions regarding roles within and contributions to the various communities in which students are members.

> *The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.*
> *(NCTM, 2000, p. 4)*

## Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

### Developing Thinking

> *K-12 Goals*
> - *thinking and learning contextually*
> - *thinking and learning creatively*
> - *thinking and learning critically.*

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This most effectively occurs through student engagement in inquiry and problem solving when students are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "What would happen if …", "Could we find …", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

### Developing Identity and Interdependence

> *K-12 Goals*
> - *understanding, valuing, and caring for oneself*
> - *understanding, valuing, and respecting human diversity and human rights and responsibilities*
> - *understanding and valuing social and environmental interdependence and sustainability.*

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accept responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others

.

## Developing Literacies

Through their mathematics learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. An important part of learning mathematical language is to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. The study of mathematics should encourage the appropriate use of technology. Moreover, students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, oral, written, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings. All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

*K-12 Goals*
- *constructing knowledge related to various literacies*
- *exploring and interpreting the world through various literacies*
- *expressing understanding and communicating meaning using various literacies.*

*Ideas are the currency of the classroom. Ideas, expressed by any participant, warrant respect and response. Ideas deserve to be appreciated and examined. Examining an idea thoughtfully is the surest sign of respect, both for the idea and its author.*
*(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, Human, 1997, p. 9)*

*Students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to a classmate or teacher, or when they formulate a question about something that is puzzling to them. Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols. Misconceptions can be identified and addressed. A side benefit is that it reminds students that they share responsibility with the teacher for the learning that occurs in the lesson.*

*(NCTM, 2000, pp. 60 – 61)*

## Developing Social Responsibility

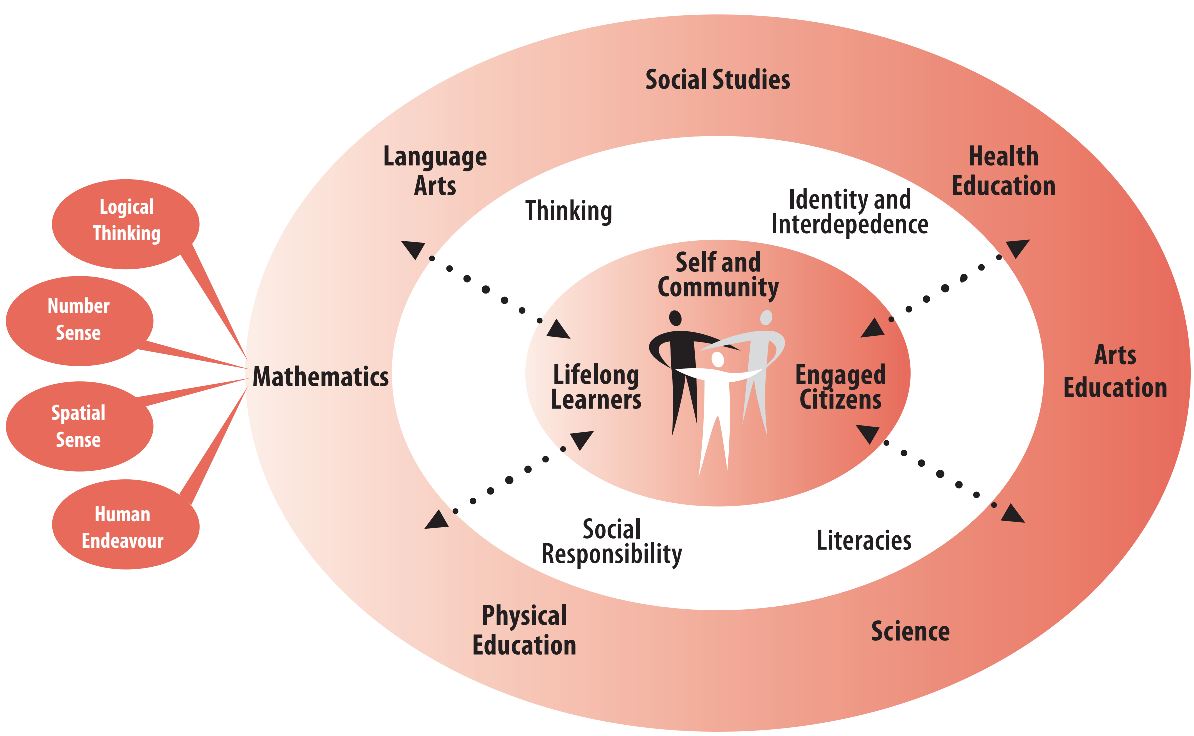
As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment be co-constructed by the teacher and students to support respectful, independent,

*K-12 Goals*
- *using moral reasoning*
- *engaging in communitarian thinking and dialogue*
- *contributing to the well-being of self, others, and the natural world.*

and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for situational validity, and strengthened.

## Aim and Goals of K-12 Mathematics

The aim of the K-12 Mathematics program is to prepare individuals who value mathematics and appreciate its role in society. The K-12 Mathematics curricula are designed to prepare students to cope confidently and competently with everyday situations that demand the use of mathematical concepts including interpreting quantitative information, estimating, performing calculations mentally, measuring, understanding spatial relationships, and problem solving. The Mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.



Defined below are four goals for K-12 Mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must, therefore, also promote student achievement with respect to the goals.

## Logical Thinking

Through their learning of K-12 Mathematics, students should **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and symbolical representations)
- conjecturing and asking "what if" (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

*A … feature of the social culture of [mathematics] classrooms is the recognition that the authority of reasonability and correctness lies in the logic and structure of the subject, rather than in the social status of the participants. The persuasiveness of an explanation, or the correctness of a solution depends on the mathematical sense it makes, not on the popularity of the presenter.*
*(Hiebert et al., 1997, p. 10)*

## Number Sense

Through their learning of K-12 Mathematics, students should **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

*As Grade 2 students learn about 100, what are some of the key understandings that students should be attaining? Why? How?*

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations

> *Students also develop understanding of place value through the strategies they invent to compute.*
> (NCTM, 2000, p. 82)

(including concrete, oral, physical, pictorial, and symbolical representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

## Spatial Sense

Through their learning of K-12 Mathematics, students should **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

> *2-D shapes are abstract ideas because they only really exist as parts of 3-D objects. How can you assess to see if your students really understand this connection?*

Development of a strong spatial sense requires students to have ongoing experiences with:
- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

> *As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop.*
> (NCTM, 2000, p. 165)

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation of conjectures based upon patterns that are discovered and tested should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

## Mathematics as a Human Endeavour

Through their learning of K-12 Mathematics, students should **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematically and personally)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet their particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

The content found within the grade level outcomes for the K-12 Mathematics programs, and its applications, is first and foremost the vehicle through which students can achieve the four goals of K-12 Mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

*What types of instructional strategies support student attainment of the K-12 Mathematics goals?*

*How can student attainment of these goals be assessed and the results be reported?*

*Meaning does not reside in tools; it is constructed by students as they use tools.*
*(Hiebert et al., 1997, p. 10)*

# Teaching Mathematics

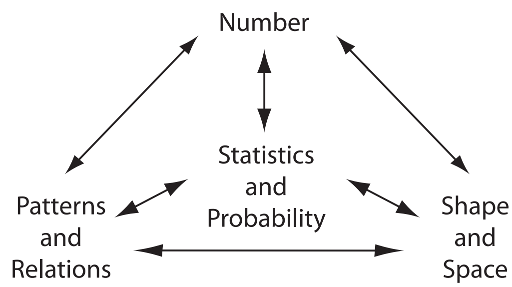
At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]." This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

## Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from covering to supporting students in discovering mathematical concepts. These facets include the organization of the outcomes into strands, seven mathematical processes, the difference between covering and discovering mathematics, the development of mathematical terminology, the continuum of understanding from the concrete to the abstract, modelling and making connections, the role of homework in mathematics, and the importance of ongoing feedback and reflection.

### Strands

The content of K-12 Mathematics can be organized in a variety of ways. In this curriculum, the outcomes and indicators are grouped according to four strands: **Number, Patterns and Relations, Shape and Space, and Statistics and Probability.**



Although this organization implies a relatedness among the outcomes identified in each of the strands, it should be noted the mathematical concepts are interrelated across the strands as well as within strands. Teachers are encouraged to design learning activities that integrate outcomes both within a strand and across the strands so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate strands.

## Mathematical Processes

This Grade 2 Mathematics curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in K-12 Mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. Teachers should consider carefully in their planning those processes indicated as being important to supporting student achievement of the various outcomes.

### Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links between their own language, ideas, and prior knowledge, the formal language and symbols of mathematics, and new learnings.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding for that terminology.

Concrete, pictorial, symbolic, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

*Communication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics.*

*(Hiebert et al., 1997, p. 6)*

*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.*
*(Caine & Caine, 1991, p.5)*

## Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

## Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

*What words would you expect to hear your Grade 2 students saying when they are estimating or using mental mathematics strategies?*

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

## Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you …?”, “Can you …?”, or “What if …?”, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

*Mathematical problem-solving often involves moving backwards and forwards between numerical/algebraic representations and pictorial representations of the problem.*
*(Haylock & Cockburn, 2003, p. 203)*

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

### Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. High-order inquiry challenges students to think and develop a sense of wonder about mathematics.

*A Grade 2 student might conjecture that the students in Grade 2 have more pet cats than dogs. What experiences and tasks will that student need to have to reach and verify such conjectures?*

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

*Posing conjectures and trying to justify them is an expected part of students' mathematical activity.*
*(NCTM, 2000, p. 191)*

### Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes including aspects such as dimensions and measurements.

*[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.*
*(Armstrong, 1993, p.10)*

Visualization is also important in the students' development of abstraction and abstract thinking and reasoning. Visualization provides a connection between the concrete, physical, and pictorial to the abstract symbolic. Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations as well as the use of communication to develop connections between different contexts, content, and representations.

*In Grade 2, there are no outcomes that explicitly require student use of technology. However, are there places where students might use technology to develop their understandings in Grade 2?*

### Technology [T]

Technology tools contribute to student achievement of a wide range of mathematical outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

*Technology should not be used as a replacement for basic understandings and intuition.*
*(NCTM, 2000, p. 25)*

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

## Discovering versus Covering

*What mathematical content in Grade 2 can students discover (through the careful planning of a teacher) and what does a teacher need to tell the students?*

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what needs to be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies and procedures, rules, and problem solving. Any learning in mathematics that is a result of the logical structure of mathematics can and should be constructed by students.

For example, in Grade 2, the students encounter increasing patterns in outcome P2.2:

> Demonstrate understanding of increasing patterns by:
> - describing
> - reproducing
> - extending
> - creating patterns using manipulatives, pictures, sounds, and actions (numbers to 100).
>
> [C, CN, PS, R, V]

In this outcome, the term "increasing patterns" is a social convention of the mathematics the students are learning and, as such, it is something that the teacher must tell the student. Analysis of the pattern in order to describe, reproduce, extend, and create increasing patterns with understanding requires students to explore, conjecture, verify, and abstract their own understandings. This type of learning requires students to work concretely, physically, orally, pictorially, in writing, and symbolically. It also requires that students share their ideas with their classmates and reflect upon how the ideas and understandings of others relate to, inform, and clarify what students individually understand. In this type of learning, the teacher does not tell the students how to do the mathematics but, rather, invites the students to explore and develop an understanding of the logical structures inherent in the mathematics in increasing patterns. Thus, the teacher's role is to create inviting and rich inquiring tasks and to use questioning to effectively probe and further students' learning.

> *The plus sign, for example, and the symbols for subtraction, multiplication, and division are all arbitrary convention. … Learning most of mathematics, however, relies on understanding its logical structures. The source of logical understanding is internal, requiring a child to process information, make sense of it, and figure out how to apply it.*
> *(Burns & Sibley, 2000, p. 19)*

## Development of Mathematical Terminology

Part of learning mathematics is learning how to speak mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that students already know or that make sense to them.

> *Teachers should model appropriate conventional vocabulary.*
> *(NCTM, 2000, p. 131)*

For example, in outcome P2.3:

> Demonstrate understanding of equality and inequality concretely and pictorially (0 to 100) by:
> - relating equality and inequality to balance
> - comparing sets
> - recording equalities with an equal sign
> - recording inequalities with a not equal sign
> - solving problems involving equality and inequality.
>
> [C, CN, R, V]

the terminology, at least in a mathematical sense, of "equality" and "inequality" will likely be new to most of the students. This does not mean, however, that students cannot use their own vocabulary to explain the relationships of equality and inequality (e.g., the same, not the same, fair share, or not fair share) that they discover through their mathematical explorations. When the students are able to describe their understandings of these two concepts, then the actual mathematical terminology is best introduced (equality and inequality) because the students have other knowledge with which to connect the new words. Upon introducing these new terms, the teacher should also be checking if the students have other connections to the new words in non-mathematical settings. For example, students may be very aware of racial, age, cultural, or gender-related inequalities. It is beneficial that students also consider such contexts to help them better understand and make connections to the new mathematical context for the words.

## The Concrete to Abstract Continuum

> *It is important for students to use representations that are meaningful to them.*
> *(NCTM, 2000, p. 140)*

It is important that, in learning mathematics, students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a concrete starting point. Therefore, the teacher must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. For example, when considering a problem about the total number of pencils, some students might find it more concrete to use pictures of pencils as a means of representing the situation. Other students might find coins more concrete because they directly associate money with the purchasing or having of a pencil.

As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following problem involving subtraction:

> Roger's mother placed 12 apples in a bowl on the centre of the table. The next day, Roger counted only 8 apples in the bowl. How many apples had been taken out of the bowl?

Depending upon how the problem is expected to be solved (or if there is any specific expectation), this problem can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures, role play), or both.

## Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and symbolic). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you write what you've done here using mathematical symbols?) or by having students compare their representations with others around the class.

> *A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.*
> *(NCTM, 2000, pp. 139)*

In making these connections, students should also be asked to reflect upon the mathematical ideas and concepts that students already know are being used in their new models (e.g., I know that addition means to put things together into a group, so I'm going to move the two sets of blocks together to determine the sum).

Making connections also involves looking for patterns. For example, in outcome N2.2:

> Demonstrate understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by:
> - representing strategies for adding and subtracting concretely, pictorially, and symbolically.
> - creating and solving problems involving addition and subtraction
> - estimating
> - using personal strategies for adding and subtracting with and without the support of manipulatives
> - analyzing the effect of adding or subtracting zero
> - analyzing the effect of the ordering of the quantities (addends, minuends, and subtrahends) in addition and subtraction statements.
>
> [C, CN, ME, PS, R, V]

the students' recognition of patterns, such as those found when adding using a hundred chart, can then be generalized into personal addition strategies to be applied and verified.

## Role of Homework

The role of homework in teaching for deep understanding is very important and also quite different from homework that is traditionally given to students. Students should be given unique problems and tasks that help students to consolidate new learnings with prior knowledge, explore possible solutions, and apply learnings to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of the drill will vary among different learners. In addition, when used as homework, drill and practice frequently serves to cause frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding of Grade 2 Mathematics, consider outcome SS2.4 :

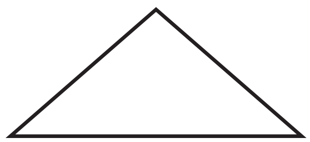
> Describe, compare and construct 2-D shapes, including:
> - triangles
> - squares
> - rectangles
> - circles.
>
> [C, CN, R, V]

*Characteristics of Good Math Homework*
- *It is accessible to children at many levels.*
- *It is interesting both to children and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to and types of math homework offered to children throughout the year.*

*(Raphel, 2000, p. 75)*

As a homework task, students might be asked to trace the tops of 6 different containers they find at home and then sort their pictures in a way that makes sense to them. In the following class, the students could work in pairs or small groups to consider questions such as:

- "How did each student sort their pictures?"
- "What is the same about some of the pictures? What is different?"
- "How might you sort all of your pictures together?"
- "I had a container at home that had a top like this:



Did any of your shapes have the same type of top? How do you know?"

The students' discussions regarding their homework can then lead into further explorations relating to outcome SS2.4, or the pictures and the discussions can be used to connect learnings from outcome SS2.4 with those from outcome SP2.1:

| |
|---|
| Demonstrate understanding of concrete graphs and pictographs. [C, CN, PS, R, V] |

with students creating their own versions of concrete graphs and pictographs to represent the data (pictures) collected.

*Feedback can take many different forms. Instead of saying, "This is what you did wrong," or "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about…?"*
*(Stiff, 2001, p. 70)*

### Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information in the teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

*Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.*
*(NCTM, 2000, p. 72)*

# Teaching for Deep Understanding

*A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences… A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.*
*(Haylock & Cockburn, 2003, p. 18)*

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, the addition sign (+) is something which the teacher must introduce and ensure that students know. It is the symbol used to show the combination or addition of two quantities. The process of adding, however, and the development of addition and subtraction facts should be discovered through the students' investigation of patterns, relationships, abstractions, and generalizations. It is important for teachers to reflect upon outcomes to identify what students need to know, understand, and be able to do. Opportunities must be provided for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

It is important that a mathematics learning environment include effective interplay of:

- reflection
- exploration of patterns and relationships
- sharing of ideas and problems
- consideration of different perspectives
- decision making
- generalizing
- verifying and proving
- modeling and representing.

*What types of things might you hear or see in a Grade 2 classroom that would indicate to you that students were learning for deep understanding?*

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

## Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are directly involved and engaged in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understandings
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning and mastery of curriculum content and skills.

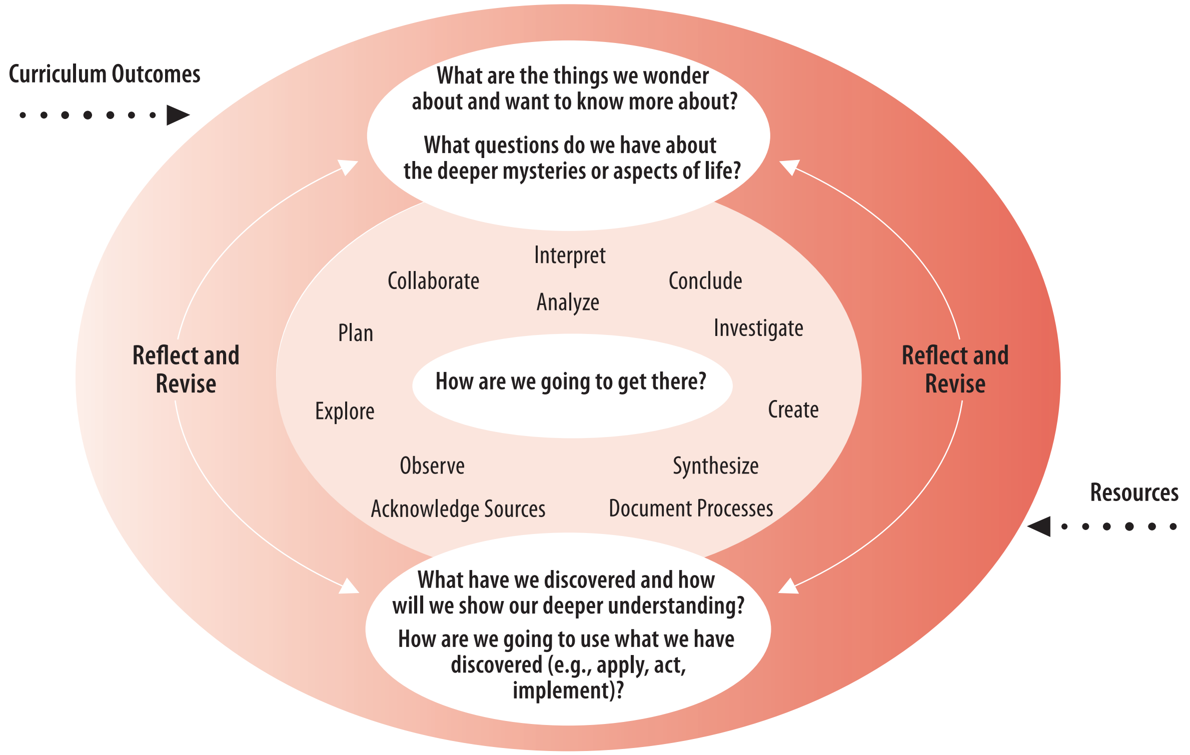
*(Adapted from Kuhlthau & Todd, 2008, p. 1)*

Inquiry learning is not a step-by-step process, but rather a cyclical process, with various phases of the process being revisited and rethought as a result of students' discoveries, insights, and co-construction of new knowledge. The following graphic shows various phases of this cyclical inquiry process.

*Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.*
*(Mills & Donnelly, 2001, p. xviii)*

*Something is only a problem if you don't know how to get from where you are to where you want to be. Be sure that Grade 2 students are solving such problems.*

## Constructing Understanding Through Inquiry




Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers will move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, conclude, document, reflect on learning, and develop new questions for further inquiry.

In Mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students too must understand this difference.

## Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in Mathematics are the key to initiating and guiding students' investigations and critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "When would you want to add two numbers less than 100?"
- "How do you know you have an answer?"
- "Will this work with every number? Every similar situation?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning and should be an integral part of planning in mathematics. Questioning should also be used to encourage students to reflect on the inquiry process and the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

*Questions may be one of the most powerful technologies invented by humans. Even though they require no batteries and need not be plugged into the wall, they are tools which help us make up our minds, solve problems, and make decisions. – Jamie McKenzie (Schuster & Canavan Anderson, 2005, p. 1)*

*Effective questions:*

- *cause genuine and relevant inquiry into the important ideas and core content.*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions.*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answers.*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons.*
- *spark meaningful connections with prior learning and personal experiences.*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

*(Wiggins & McTighe, 2005, p. 110)*

*As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.*
*(Schuster & Canavan Anderson, 2005, p. 1)*

- *help students make sense of the mathematics.*
- *are open-ended, whether in answer or approach. There may be multiple answers or multiple approaches.*
- *empower students to unravel their misconceptions.*
- *not only require the application of facts and procedures but encourage students to make connections and generalizations.*
- *are accessible to all students in their language and offer an entry point for all students.*
- *lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.*

*(Schuster & Canavan Anderson, 2005, p. 3)*

## Reflection and Documentation of Inquiry

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, drafts, models, and works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

It is important that students are required, and allowed to engage in, the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

*When we ask good questions in math class, we invite our students to think, to understand, and to share a mathematical journey with their classmates and teachers alike. Students are no longer passive receivers of information when asked questions that challenge their understandings and convictions about mathematics. They become active and engaged in the construction of their own mathematical understanding and knowledge.*
*(Schuster & Canavan Anderson, 2005, p. 1)*

# Outcomes and Indicators

<table>
<tr><th colspan="2">Number</th></tr>
<tr><td colspan="2">Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour</td></tr>
<tr><td>Outcomes (What students are expected to know and be able to do.)</td><td>Indicators (Students who have achieved this outcome should be able to:)</td></tr>
<tr><td>N2.1 Demonstrate understanding of whole numbers to 100 (concretely, pictorially, physically, orally, in writing, and symbolically) by:<br>• representing (including place value)<br>• describing<br>• skip counting<br>• differentiating between odd and even numbers<br>• estimating with referents<br>• comparing two numbers<br>• ordering three or more numbers.<br>[C, CN, ME, PS, R, V]</td><td>a. Describe the patterns related to quantity and place value of adjacent digit positions moving from right to left within a whole number.<br><br>b. Describe the meaning of quantities to 100 by relating them to self, family, or community and explain what effect each successive numeral position has on the actual quantity.<br><br>c. Pose and solve problems that explore the quantity of whole numbers to 100 (e.g., a student might wonder: “How many pets would there be if everyone in the class brought their pets to class”).<br><br>d. Represent quantities to 100 using proportional materials (e.g., tallies, ten frames, and base ten blocks) and explain how the representation relates to the numeral used to represent the quantity.<br><br>e. Represent quantities to 100 using non-proportional materials (e.g., stir sticks and popsicle sticks, and coins) and explain how the representation relates to the numeral used to represent the quantity.<br><br>f. Identify whole numbers to 100 stated as a numeral or word form in everyday situations and read the number out loud (e.g., 24 on the classroom door would be read as twenty-four, and read out loud “seventy-three” when found in a piece of writing being read in class).<br><br>g. Create different decompositions for a given quantity using concrete manipulatives or pictures and explain orally how the different decompositions represent the original quantity.<br><br>h. Write numbers to twenty in words when said out loud or given as a numeral.<br><br>i. Analyze a sequence of numbers in order to describe the sequence in terms of a skip counting strategy (by 2s, 5s, or 10s as well as forward and backward) and extend the sequence using the pattern.</td></tr>
</table>

| Outcomes | Indicators |
| --- | --- |
| ***N2.1 (continued)*** | j. Analyze an ordered number sequence (including a hundred chart) for errors or omissions and explain the reasoning.<br>k. Sort a set of personally relevant numbers into odd and even numbers.<br>l. Hypothesize and verify strategies for skip counting by 10s beginning at any whole number from 0 to 9 (e.g., in a hundred chart, the skip counted numbers always lie on a vertical line; using base ten blocks, skip counting by 10s always increases the number of rods by one; or using numerals, the tens place value always increases by 1 (meaning 10) when skip counting by 10s forwards).<br>m. Order a set of personally relevant numbers in ascending or descending order and verify the resulting sequence (e.g., using a hundred chart, number line, ten frames, or place value).<br>n. Analyze a number relevant to one's self, family, or community to determine if it is odd or even and verify the conclusion by using concrete, pictorial, or physical representations.<br>o. Estimate a quantity from one's life, family, or community by using a referent (known quantity), including 10, and explain the strategies used.<br>p. Select a referent for determining a particular quantity and explain the choice.<br>q. Critique the statement "A referent for 10 is always a good referent to use".<br>r. Represent a 2-digit numeral using ten frames or other proportional base ten materials.<br>s. Create representations of different decompositions of the same quantity and explain how the representations represent the same amount.<br>t. Explain, using concrete or pictorial representations, the meaning of each digit within a 2-digit numeral with both digits the same (e.g., for the numeral 22, the first digit represents two tens - twenty counters - and the second digit represents two ones - two counters).<br>u. Defend the statement "The value of a digit depends on its placement within a numeral".<br>v. Demonstrate how to count objects using groupings of 10s and 1s and explain how those groups help in the writing of the 2-digit number that represents the quantity of objects. |

*Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
|---|---|
| ***N2.2** Demonstrate understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by:*<br>• *representing strategies for adding and subtracting concretely, pictorially, and symbolically*<br>• *creating and solving problems involving addition and subtraction*<br>• *estimating*<br>• *using personal strategies for adding and subtracting with and without the support of manipulatives*<br>• *analyzing the effect of adding or subtracting zero*<br>• *analyzing the effect of the ordering of the quantities (addends, minuends, and subtrahends) in addition and subtraction statements.*<br>*[C, CN, ME, PS, R, V]* | a. Generalize rules for adding when one addend is zero and for subtracting zero from a quantity and use concrete, pictorial, physical, or oral models to explain the reasoning.<br>b. Verify rules generalized for addition and subtraction involving a quantity of zero.<br>c. Model concretely, pictorially, or physically situations that involve the addition or subtraction of 1 and 2-digit numbers (with answers to 100) and explain how to record the process shown in the model symbolically.<br>d. Generalize and apply strategies for adding and subtracting 1 and 2-digit numbers (with answers to 100).<br>e. Create, model symbolically (and concretely, pictorially, or physically if desired), and solve addition and subtraction problems related to situations relevant to one's self, family, or community.<br>f. Critique the statement "You can add or subtract numbers in any order and still get the same answer" and provide examples to support the critique.<br>g. Select and explain a mental mathematics strategy that can be used to determine a sum of up to 18 (or related difference):<br>• doubles (e.g., for 4 + 6, think 5 + 5)<br>• doubles plus one (e.g., for 4 + 5, think 4 + 4 + 1)<br>• doubles take away one (e.g., for 4 + 5, think 5 + 5 – 1)<br>• doubles plus two (e.g., for 4 + 6, think 4 + 4 + 2)<br>• doubles take away two (e.g., for 4 + 6, think 6 + 6 – 2)<br>• making 10 (e.g., for 7 + 5, think 7 + 3 + 2)<br>• building on a known double (e.g., 6 + 6 = 12, so 6 + 7 = 12 + 1 = 13). |

# Mathematics 2

## Patterns and Relations

*Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
| --- | --- |
| ***P2.1** Demonstrate understanding of repeating patterns (three to five elements) by:*<br>• *describing*<br>• *representing patterns in alternate modes*<br>• *extending*<br>• *comparing*<br>• *creating patterns using manipulatives, pictures, sounds, and actions.*<br>*[C, CN, PS, R, V]* | a. Identify and describe repeating patterns found in familiar situations and justify why the descriptions are those of repeating patterns (e.g., "Every day I get up, brush my hair, wash my face, have breakfast" - this is a repeating pattern because I do the same pattern over and over again).<br>b. Analyze a repeating pattern to identify the core of the pattern.<br>c. Analyze a repeating pattern for its core and extend the pattern so the core appears twice more.<br>d. Analyze an intended repeating pattern to identify possible errors.<br>e. Create a repeating pattern and explain the reasoning.<br>f. Predict an upcoming element in a repeating pattern and verify the prediction.<br>g. Analyze two repeating patterns that are represented using different materials or modes (e.g., a diagram of a repeating pattern with a core of red, red, blue, blue, blue and a sound pattern with a core of buzz, buzz, snap, snap, snap) and present ways in which the patterns are related (e.g., there are two different elements in the core of each pattern, and the core pattern is element 1, element 1, element 2, element 2, element 2 in both patterns). |

*Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
| --- | --- |
| ***P2.2** Demonstrate understanding of increasing patterns by:*<br>• *describing*<br>• *reproducing*<br>• *extending*<br>• *creating patterns using manipulatives, pictures, sounds, and actions (numbers to 100).*<br>*[C, CN, PS, R, V]* | a. Identify and describe increasing patterns in familiar situations (e.g., hundred chart, number line, addition tables, calendar, a tiling pattern or drawings, apartment numbers, years, or age).<br>b. Analyze a numerical increasing pattern for its pattern rule and extend the pattern.<br>c. Analyze a non-numerical increasing pattern and extend the pattern.<br>d. Reproduce an increasing numerical pattern using an alternate form (e.g., sound, action, concrete objects, or diagrams) and explain the reasoning. |

| Outcomes | Indicators |
| --- | --- |
| ***P2.2 (continued)*** | e. Reproduce a concrete or pictorial increasing pattern using numbers and explain the reasoning.<br>f. Solve problems involving increasing patterns (e.g., determine the house number for a particular house given the house numbers for the other homes on the block, or determining the number of cubes in the missing structure) and explain the reasoning.<br>g. Create an increasing pattern, represent the pattern in different modes (using manipulatives, diagrams, sounds, actions, and/or physical movements), and explain the pattern rule. |

*Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
| --- | --- |
| ***P2.3 Demonstrate understanding of equality and inequality concretely and pictorially (0 to 100) by:***<br>• ***relating equality and inequality to balance***<br>• ***comparing sets***<br>• ***recording equalities with an equal sign***<br>• ***recording inequalities with a not equal sign***<br>• ***solving problems involving equality and inequality.***<br>***[C, CN, R, V]*** | a. Compare two quantities of the same object (same shape and mass) by using a balance scale to determine if the quantities are equal or not.<br>b. Construct two unequal sets using identical objects and verify orally and concretely that the sets are not equal.<br>c. Analyze the impact of changing one of two equal sets upon the equality of the two sets.<br>d. Analyze the impact of making changes (equal and unequal) to both of two equal sets upon the equality of the sets.<br>e. Analyze and sort sets according to equality and explain the reasoning.<br>f. Model two number expressions to determine if the expressions are equal (=) or not equal (≠) and write a number sentence to show the relationship (e.g., 3 + 2 and 4 + 1 are both equal to 5, so the two expressions are = and I write 3 + 2 = 4 + 1; 7 - 5 and 3 are not the same quantity, so I write 7 – 5 ≠ 3).<br>g. Create statements of equality and non-equality and model the statements to verify the relationship. |

## Shape and Space

*Goals: Spatial Sense, Logical Thinking, Number Sense, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
|---|---|
| ***SS2.1** Demonstrate understanding of non-standard units for linear measurement by:*<br>• ***describing the choice and appropriate use of non-standard units***<br>• ***estimating***<br>• ***measuring***<br>• ***comparing and analyzing measurements.***<br>*[C, CN, ME, R, V]* | a. Defend the choice of a non-standard unit for measuring a length in a situation relevant to one's self, family, or community.<br>b. Estimate a personally relevant length, including the distance around a space, using one's own choice of standard unit.<br>c. Compare estimates of the same length made by different units and provide reasons for different values being stated for the measurements.<br>d. Critique the statement "It is possible to get an exact length measurement".<br>e. Devise and apply strategies for determining estimates for linear and non-linear lengths using non-standard units.<br>f. Explain why overlapping or leaving gaps does not result in accurate measurements.<br>g. Explain why the same non-standard unit should be used to determine length measurements that are to be compared.<br>h. Compare and order sets of related objects, possibly including people, according to a length measurement. |

*Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
|---|---|
| ***SS2.2** Demonstrate understanding of non-standard units for measurement of mass by:*<br>• ***describing the choice and appropriate use of non-standard units***<br>• ***estimating***<br>• ***measuring***<br>• ***comparing and analyzing measurements.***<br>*[C, CN, ME, R, V]* | a. Defend the choice of a non-standard unit for measuring a mass in a situation relevant to one's self, family, or community.<br>b. Estimate the mass of a personally relevant object using one's own choice of standard unit.<br>c. Identify a non-standard unit for measuring mass that would not be a good choice in a particular situation and explain the reasoning (e.g., to measure the mass of a desk, it would not make sense to use an eraser as the standard unit because a desk has so much more mass than an eraser and so it would take too many erasers, or to measure the mass of a library book using the standard unit of a student in the class because the student already has a greater mass than the book).<br>d. Compare estimates of the mass of the same object determined using different standard units and provide reasons for different values being stated for the measurements. |

| Outcomes | Indicators |
|---|---|
| ***SS2.2 (continued)*** | e. Explain why the same non-standard unit should be used to determine mass measurements that are to be compared.<br>f. Compare and order sets of related objects according to mass measurements and explain the reasoning. |

*Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
|---|---|
| ***SS2.3 Describe, compare, and construct 3-D objects, including:***<br>• ***cubes***<br>• ***spheres***<br>• ***cones***<br>• ***cylinders***<br>• ***pyramids.***<br>*[C, CN, R, V]* | a. Identify examples of cubes, spheres, cones, cylinders, and pyramids as found in the classroom, home, and community.<br>b. Sort a set of personally relevant 3-D objects and explain the sorting rule used.<br>c. Compare the attributes of cubes, spheres, cones, cylinders, and pyramids and generalize descriptions of each category of 3-D objects.<br>d. Compare two 3-D objects of the same type (e.g., both are cylinders) and explain how the dimensions of the objects can be used to compare the objects (one-to-one correspondence or non-standard units).<br>e. Compare two 3-D objects in different orientations (e.g., "If I was to flip this object over, the two objects would have the same height.").<br>f. Create and describe a concrete representation of a personally relevant 3-D object.<br>g. Sort 3-D objects according to two attributes and explain the sorting rule used. |

*Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
|---|---|
| ***SS2.4** Describe, compare, and construct 2-D shapes, including:*<br>• *triangles*<br>• *squares*<br>• *rectangles*<br>• *circles.*<br>*[C, CN, R, V]* | a. Identify examples of triangles, rectangles, squares, and circles as found in personal experiences.<br>b. Compare the attributes of triangles, squares, rectangles, and circles and generalize descriptions of each category of 2-D shapes objects.<br>c. Critique the statement "A 2-D shape can either be a rectangle or a square, but not both".<br>d. Compare two 2-D shapes of the same type (e.g., both are circles) and explain how the dimensions of the shapes can be used to compare the shapes (one-to-one correspondence or non-standard units).<br>e. Classify 2-D shapes arranged in different orientations according to the type (triangle, rectangle, square, or circle) and explain the impact of the orientation of shape on its classification.<br>f. Create a model to represent a 2-D shape.<br>g. Sort regular and irregular 2-D shapes according to two attributes and explain the sorting rule used. |

*Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
|---|---|
| ***SS2.5** Demonstrate understanding of the relationship between 2-D shapes and 3-D objects.*<br>*[C, CN, R, V]* | a. Analyze the differences between two pre-sorted sets of objects and/or pictures of shapes and explain how the objects and shapes were sorted.<br>b. Analyze a set of objects and/or pictures of shapes to identify two common attributes of each member of the set.<br>c. Describe the faces of a personally relevant 3-D object by comparing the faces to 2-D shapes (such as triangles, squares, rectangles, or circles).<br>d. Analyze (using concrete models of 3-D objects) a set of descriptions of the 2-D faces of a 3-D object to identify the 3-D object (e.g., "A 3-D object has one rectangular face and four triangular faces – what type of object is it?" "A pyramid.").<br>e. Analyze and correct the statement "The tissue box is a rectangle". |

## Statistics and Probability

*Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour*

| Outcomes | Indicators |
| --- | --- |
| ***SP2.1 Demonstrate understanding of concrete graphs and pictographs. [C, CN, PS, R, V]*** | a. Formulate a question relevant to one's self, family, or community that can be answered by gathering information from people.<br>b. Select an organizational structure, such as sets of concrete objects, tallies, checkmarks, charts, or lists, for the collection of data that are gathered.<br>c. Pose questions related to gathered data and explain how the data can be used to answer those questions.<br>d. Analyze concrete graphs to identify and define the common attributes of a concrete graph.<br>e. Analyze pictographs to identify and define the common attributes of a pictograph.<br>f. Create a concrete graph to display collected data and make and support conclusions based upon the graph.<br>g. Create a pictograph (using one-to-one correspondence) to display collected data and make and support conclusions based on the graph.<br>h. Create and solve a problem for which data can be collected from individuals in the class, at home, in the school, or within the community and give a presentation of how the collection, organization, display, and analysis of data were done to attain a solution to the problem. |

## Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- ☑ Achievement of provincial curriculum outcomes
- ☑ Effectiveness of teaching strategies employed
- ☑ Student self-reflection on learning.

*Assembling evidence from a variety of sources is more likely to yield an accurate picture.*
*(NCTM, 2000, p. 24)*

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be based on the achievement of curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Grade 2 Mathematics. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

*Assessment should not merely be done to students; rather it should be done for students.*
*(NCTM, 2000, p. 22)*

**Assessment for learning** involves the use of information about student progress to support and improve student learning and inform instructional practices and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

*What are examples of assessments as learning that could be used in Grade 2 mathematics and what would be the purpose of those assessments?*

**Assessment as learning** involves student reflection on and monitoring of her/his own progress and:

- students self-reflect and critically analyze learning related to curricular outcomes without anxiety or censure
- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

**Assessment of learning** involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

*Assessment should become a routine part of the ongoing classroom activity rather than an interruption.*
*(NCTM, 2000, p. 23)*

In mathematics, students need to be regularly engaged in assessment as learning. The assessments used should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the mathematical content for Grade 2.

## Connections with Other Areas of Study

There are many possibilities for connecting Grade 2 mathematical learning with the learning occurring in other subject areas. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. Following are just a few of the ways in which mathematics can be connected to other subject areas (and other subject areas connected to mathematics) at Grade 2.

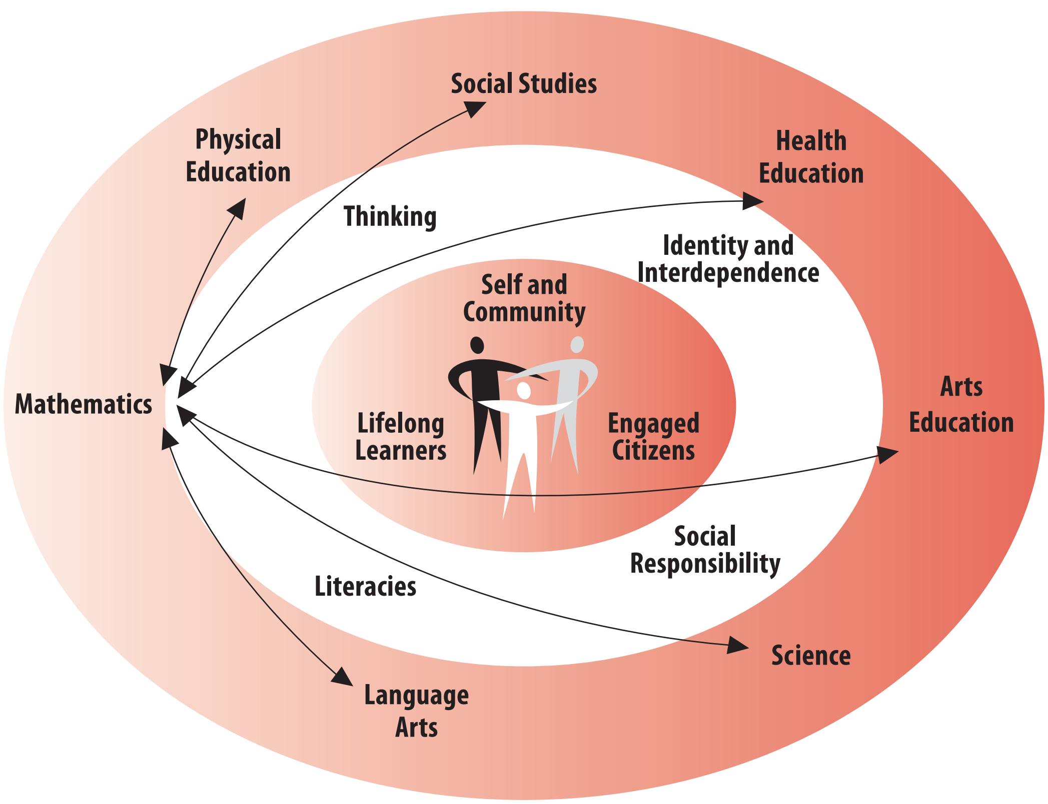
**Arts Education** – One of the strongest connections that can be made between Arts Education and Mathematics is through the creation and use of multiple representations. As students in Grade 2 explore patterns, numbers to 100, 3-D objects and 2-D shapes, and concrete graphs, the use of physical movement and sounds as well as the exploration of different concrete and pictorial representations of the mathematical content allows students to understand mathematics through processes and concepts the students are learning about in Arts Education. For example, outcome P2.1:

Demonstrate understanding of repeating patterns (three to five elements) by:

- describing
- representing patterns in alternate modes
- extending
- comparing
- creating patterns using manipulatives, pictures, sounds, and actions.

[C, CN, PS, R, V]

can be explored by students when they are also learning about how music, rhythms, or dance phrases, which can be repeated, also have a beginning and end, just like the core of a repeating pattern (e.g., the pattern ABBABBABB has a core that begins with A and ends with B). Students could, therefore, use music or dance phrases to represent patterns being explored in mathematics. Students could also observe repeating patterns in First Nations' beading and dance regalia, or analyse how shapes, lines, colours, and textures are used to create repeating patterns in quilts, clothing designs, architecture, and other works of visual art. Alternatively, in arts education, students could describe patterns in arts expressions using alternate modes (such as sounds, letters, or concrete objects) as they would in their Mathematics classes.



**English Language Arts** (ELA) – ELA and Mathematics share a common interest in students developing their abilities to reflect upon and communicate about their learnings through viewing, listening, reading, representing, speaking, and writing. As an example of how Mathematics involves these strands of language consider outcome N2.1:

> Demonstrate understanding of whole numbers to 100 (concretely, pictorially, physically, orally, in writing, and symbolically) by:
> - representing (including place value)
> - describing
> - skip counting
> - differentiating between odd and even numbers
> - estimating with referents
> - comparing two numbers
> - ordering three or more numbers.
>
> [C, CN, ME, PS, R, V]

To achieve this learning outcome, students are to be explicitly engaged in understanding the nature and relationships of whole numbers to 100. In developing such understandings, students first need to create and relate different representations for whole numbers from the students' personal experiences, such as numbers said at home, numbers within a text from another class, or numbers embedded within mathematical problems students read. As the students develop their ability to read, they will also be able to engage in solving written problems involving whole numbers. Students would share those representations with their classmates and view the representations created by others.

Through the use of language, both orally and in writing, students should communicate with their classmates and others about their developing understandings of whole numbers and seek clarification of the ideas presented by other students. The students would be engaged in responding to what is presented to them and to questions asked of what has been presented. Such activity requires students to effectively speak, listen, and show appreciation for the ideas communicated by other students. By actively engaging in the use of language and other ways of representing their understanding, students will reflect deeply upon their learnings, leading to a combination of affirmations, changes, and extensions to each student's understandings of whole numbers to 100.

**Health Education** – At Grade 2, most of the connections that can be made between Mathematics and Health Education are related to the Mathematics goal of understanding Mathematics as a Human Endeavour. In Grade 2 Health Education, students learn about everyone being special, unique, and able to succeed and how communities benefit from this diversity. This is also a major understanding within the Mathematics as a Human Endeavour goal which is related to every outcome in Grade 2 Mathematics. Thus, as students engage in mathematical inquiries and share their understandings with their classmates, the understandings they are learning in Health Education with respect to the importance and value of the contributions of everyone are also to be emphasized and respected.

As well, much of the learnings in Health Education can be used as contexts for the students' understandings related to outcome SP2.1:

> Demonstrate understanding of concrete graphs and pictographs. [C, CN, PS, R, V]

As an example, as the students explore Canada's Food Guide and making their own snack choices, the students could regularly collect data from the class regarding the types of snack choices being made and create a series of concrete graphs or pictographs to record the data and the ongoing process. Questions such as, "What was the impact of Hallowe'en on the overall healthy snack choices?" will naturally emerge for discussion both in Mathematics and in Health Education from the data collected and represented as the students construct their understandings related to outcome SP2.1.

**Physical Education** – There are many opportunities for teachers to create learning experiences that connect Physical Education and Mathematics. These learning experiences can provide students the opportunity to expand both their mathematical and physical skills and understandings. Whether students are learning or practising what they have learned about quantities to 100, adding and subtracting whole numbers, patterns, linear measurement using non-standard units, or concrete graphs and pictographs, students can also be engaged in working towards their skillful performance of locomotor, non-locomotor, and manipulative skills. For example, consider how these locomotor, non-locomotor, and manipulative skills can be used in Mathematics to help students develop a deeper understanding of outcome SS2.1:

Demonstrate understanding of non-standard units for linear measurement by:
- describing the choice and appropriate use of non-standard units
- estimating
- measuring
- comparing and analyzing measurements.

[C, CN, ME, R, V]

The skills from Physical Education, such as skipping, sliding, jumping, hopping, or bouncing a ball, can be incorporated as forms of non-standard units. Questions like:
- “How many skips do you think it would take to skip down the hall?”
- “Will it take more or less hops than skips to go across the classroom and why?”
- “How could you adjust your locomotor movement (e.g., skips or hops) to decrease the number of movements needed to measure the length of the room?”

engage students in their development of skillful performance of the movements. Such questions also provide relevant contexts in which students can demonstrate their understandings of estimating and measuring linear measurements and participate in an inquiry and understanding of how the length of the unit of measure impacts the quantity of the measurement. As students carry out such explorations, it is important to compare the results to those of their classmates, providing additional learning opportunities for teachers to help students develop a sense of a need for common measurement units.

Similarly, activities within a Physical Education class could be used to strengthen the students’ mathematical understandings of measurement. When students in Grade 2 are creating new target games, students can also be engaged in using their measuring and estimating skills in designing the target for the game as well as the distances over which objects are to be moved to the target. Students can share their strategies for determining the measurements during their Mathematics class and pictures taken of the different targets and game set-ups can be used to engage the students in comparing the measurements used and how they were determined.

**Science** – Mathematics and Science have many common topics and features, such as the recognition and description of patterns, sorting and categorizing, measurement, and the use of multiple representations. Opportunities for connections occur in the Grade 2 Science topics of Animal Growth and Changes and Relative Position and Motion. In Relative Position and Motion, students are engaged in measuring and describing the relative position of themselves and 3-D objects in the physical world. Thus, the students' mathematical learnings can support their Science studies, and their Science studies can provide interesting and relevant contexts for their mathematical investigations into measurement and other topics.

In Animal Growth and Changes, students of Grade 2 Science are comparing and classifying familiar animals according to specific types of characteristics or attributes. This parallels outcome SS2.3:

> Describe, compare, and construct 3-D objects, including:
> - cubes
> - spheres
> - cones
> - cylinders
> - pyramids.
>
> [C, CN, R, V]

During the students' explorations of 3-D objects, within which animals could be considered, students need to recognize distinguishing attributes and sort the 3-D objects according to selected attributes. In Mathematics, the use of animals and animal characteristics that the students are already familiar with (wings or no wings, number of limbs, colour) allows the students the opportunity to use prior knowledge to develop an understanding of the identification of characteristics and sorting. These understandings will help students to successfully attain these aspects of their Grade 2 learnings in both Mathematics and Science and to help make connections between processes and strategies regularly used within the two disciplines.

**Social Studies** – Social Studies and Mathematics often connect through the investigation of patterns and trends and in the representation of data. In Grade two, students in Social Studies are focusing on understanding communities. In doing so, students learn to identify critical attributes of communities and to sort and categorize communities based on those attributes. This study of attributes and categorizations can be linked to comparing, sorting, and categorizing 3-D objects and 2-D shapes. In addition, students in Grade 2 Social Studies consider different ways to represent the diverse cultural groups within different communities. Within their study of outcome SP2.1:

> Demonstrate understanding of concrete graphs and pictographs. [C, CN, PS, R, V]

students can use the data collected regarding the cultural groups within different communities and choose pictures or concrete materials that are representative of those cultures to produce a concrete graph or pictograph of the community's cultural diversity. It should be noted that the communities considered will have to be scrutinized by the teacher to ensure that the quantities do not exceed 100 people because of the students' development of their understanding of whole numbers. Students will not be able to create concrete graphs or pictographs for larger quantities, but such displays that have been created using technology or found within resource materials can be used to ask students interpretation questions.

## Glossary

**Addend:** Any quantity being added to another quantity (e.g., in the expression 32 + 57, both 32 and 57 are addends).

**Attributes:** Characteristics of 2-D shapes and 3-D objects that can be used to compare and sort sets of 2-D shapes and 3-D objects (e.g., colour, relative size, number of corners, number of lines of symmetry).

**Benchmarks:** Numeric quantities used to compare and order other numeric quantities. For example, 0, 5, 10, and 20 are often used as benchmarks when placing whole numbers on a number line.

**Concrete Graph:** A graph which uses actual objects to show how often something occurs.

**Correspondence (one-to-one):** A correspondence is a description of how one set of numbers (or objects) is mapped to a second set of objects. For example, a correspondence might describe how individual students can sit in a given number of chairs. One-to-one correspondence can also be used to determine if there are enough, too many, or just the right number of apples in order for each child to have exactly one apple.

**Equality as a Balance and Inequality as Imbalance:** The equal sign represents the idea of equivalence. For many students, it means do the question. For some students, the equal sign in an expression such as 2 + 5 = means to add. When exploring equality and inequality, by using objects on a balance scale, students discover the relationships between and among the mass of the objects. The equal sign in an equation is like a scale: both sides, left and right, must be the same in order for the scale to stay in balance and the equation to be true. When the scale is imbalanced, the equation is not true. Using $2 + 5 = \square$, rather than simply 2 + 5 = helps students understand that the equal sign (=) represents equality rather than "do the work" or "do the question".

**Interdisciplinary:** Disciplines connected by common concepts and skills embedded in disciplinary outcomes.

**Minuend:** In a subtraction sentence, the quantity that is being decreased (e.g., in the subtraction sentence 84 – 55, 84 is the minuend).

**Multidisciplinary:** Discipline outcomes organized around a theme and learned through the structure of the disciplines.

**Number, Numeral, Digit:** A number is the name that we give to quantities. For example, there are 7 days in a week, or I have three brothers – both seven and three are numbers in these situations because they are defining a quantity. The symbolic representation of a number, such as 287, is called the numeral. If 287 is not being used to define a quantity, we call it a numeral.

Numerals, as the symbolic representation of numbers, are made up of a series of digits. The Hindu-Arabic number system that we use has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note: sometimes students are confused between these digits and their finger digits – this is because they count their fingers starting at one and get to ten rather than zero to nine.) These digits are also numerals and can be numbers (representing a quantity), but all numbers and all numerals are combinations of digits. The placement of a digit in a number or numeral affects the place value of the digit and, hence, how much of the quantity that it represents. For example, in 326, the 2 is contributing 20 to the total, while in 236 the 2 contributes 200 to the total quantity.

**Object:** Object is used to refer to a three-dimensional geometrical figure. To distinguish this meaning from that of shape, the word "object" is preceded by the descriptor "3-D".

**Personal Strategies:** Personal strategies are strategies that the students have constructed and understand. Outcomes and indicators that specify the use of personal strategies convey the message that there is not a single procedure that is correct. Students should be encouraged to explore, share, and make decisions about what strategies to use in different contexts. Development of personal strategies is an indicator of the attainment of deeper understanding.

**Pictograph:** A graph which uses pictures or symbols to show how often something occurs.

**Referents:** A concrete approximation of a quantity or unit of measurement. For example, seeing what 25 beans in a container looks like makes it possible to estimate the number of beans the same container will hold when it is full of the same kind of beans. Compensation must be made if the container is filled with smaller or larger beans than the referent or if the size or shape of the container is changed.

**Representations:** Mathematical ideas can be represented and manipulated in a variety of forms including concrete manipulatives, visual designs, sounds and speech, physical movements, and symbolic notations (such as numerals and operation signs). Students need to have experiences in working with many different types of representations, and in transferring and translating knowledge between the different forms of representations.

**Shape:** In this curriculum, shape is used to refer to two-dimensional geometric figures and is thus preceded by "2-D". The term shape is sometimes also used in mathematics resources and conversations to refer to three-dimensional geometric figures. It is important that students learn to be clear in identifying whether their use of the term shape is in reference to a 2-D or 3-D geometrical figure.

**Subtrahend:** In a subtraction statement, the quantity that is being subtracted (e.g., in the subtraction statement 90 – 26, 26 is the subtrahend).

**Transdisciplinary:** All knowledge interconnected and interdependent; real-life contexts emphasized and investigated through student questions.

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# Feedback Form

The Ministry of Education welcomes your response to this curriculum and invites you to complete and return this feedback form.

Document Title: **Mathematics Grade 2 Curriculum**

1. Please indicate your role in the learning community

☐ parent ☐ teacher ☐ resource teacher

☐ guidance counsellor ☐ school administrator ☐ school board trustee

☐ teacher librarian ☐ school community council member

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What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

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☐ online

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3. How does this curriculum address the needs of your learning community or organization? Please explain.

4. Please respond to each of the following statements by circling the applicable number.

| The curriculum content is: | Strongly Agree | Agree | Disagree | Strongly Disagree |
|---|---|---|---|---|
| a. appropriate for its intended purpose | 1 | 2 | 3 | 4 |
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| c. clear and well organized | 1 | 2 | 3 | 4 |
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| e. informative | 1 | 2 | 3 | 4 |

5. Explain which aspects you found to be:

   Most useful:

   Least useful:

6. Additional comments:

7. Optional:

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