Ministry of Education

Saskatchewan Curriculum

Mathematics

3

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Introduction

As a Required Area of Study, mathematics is to be allocated 210 minutes per week for the entire school year at Grade 3. It is important that students receive the full amount of time allocated to their mathematical learning and that the learning be focused upon students attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

The outcomes in Grade 3 Mathematics build upon students' prior learnings and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These continuing learnings prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

Outcomes describe the knowledge, skills, and understandings that students are expected to attain by the end of a particular grade level.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to know and/or be able to do in order to achieve the learnings intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can be created by teachers to meet the needs and circumstances of their students and communities.

Indicators are a representative list of the types of things a student should know or be able to do if they have attained the outcome.

Within the outcomes and indicators in this curriculum, the terms "including" and "such as", as well as the abbreviation "e.g.," occur. The use of each term serves a specific purpose. The term "including" prescribes content, contexts, or strategies that students must experience in their learning, without excluding other possibilities. For example, an indicator might say that students are to describe 3-D objects, including rectangular prisms. This would mean that, although other 3-D objects could be explored and described, it is mandatory that rectangular prisms be included.

The term "such as" provides examples of possible broad categories of content, contexts, or strategies that teachers or students may choose, without excluding other possibilities. For example, an indicator might include the phrase "such as concrete materials, pictures, and symbolic decompositions" as examples of the types of representations that might be used by students. This statement provides teachers and students with possible categories of representations to consider, while not excluding other forms.

Finally, the abbreviation "e.g.," offers specific examples of what a term, concept, or strategy might look like. For example, an indicator might include the phrase "e.g., a hundred chart or number line" which are specific types of concrete and pictorial representations that a student might choose to use.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Grade 3 Mathematics outcomes are based on the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K-9 Mathematics* outcomes (2006).

Changes throughout all of the grades of Mathematics have been made for a number of reasons including:

- decreasing content in each grade to allow for more depth of understanding
- rearranging concepts to allow for greater depth of learning in one year and to align related mathematical concepts
- increasing the focus on numeracy (i.e., understanding numbers and their relationship to each other) beginning in Kindergarten
- introducing algebraic thinking earlier.

In Grade 3, students begin learning about the concepts of multiplication and division.

Also included in this curriculum is information regarding how Grade 3 Mathematics connects to the K-12 goals for mathematics. These goals define the purposes of mathematics education for Saskatchewan students.

In addition, teachers will find discussions of the critical characteristics of mathematics education, assessment and evaluation of student learning in mathematics, inquiry in mathematics, questioning in mathematics, and connections between Grade 3 Mathematics and other Grade 3 areas of study within this curriculum.

Finally, the Glossary provides explanations of some of the mathematical terminology used in this curriculum.

Core Curriculum

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.

(NCTM, 2000, p. 4)

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its various components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* on the Ministry of Education website.

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

Developing Lifelong Learners

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

Developing lifelong learners is related to the following Goals of Education:

- *Basic Skills*
- *Lifelong Learning*
- *Self Concept Development*
- *Positive Lifestyle.*

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 Mathematics). It is crucial that the students discover the mathematics outlined in the curriculum rather than the teacher covering it.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
(NCTM, 2000, p. 20)

Developing a Sense of Self and Community

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue are exposed to a wide variety of perspectives and strategies from which to construct a personalized deep understanding of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When

Developing a sense of self and community is related to the following Goals of Education:

- *Understanding & Relating to Others*
- *Self Concept Development*
- *Positive Lifestyle*
- *Spiritual Development.*

Many of the topics and problems in a mathematics classroom can be initiated by the children themselves. In a classroom focused on working mathematically, teachers and children work together as a community of learners; they explore ideas together and share what they find. It is very different to the traditional method of mathematics teaching, which begins with a demonstration by a teacher and continues with children practicing what has been demonstrated.
(Skinner, 1999, p. 7)

encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Mathematics provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students develop their personal and social identity, and learn healthy and positive ways of interacting and working together with others.

Developing engaged citizens is related to the following Goals of Education:
- *Understanding & Relating to Others*
- *Positive Lifestyle*
- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change.*

Developing Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students such as trends in climate change, homelessness, health issues (hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students derive through mathematical analysis, they become better informed and have a greater respect for and understanding of differing opinions and possible options. With these understandings, students can make better informed and more personalized decisions regarding roles within, and contributions to, the various communities in which students are members.

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

K-12 Goals for Developing Thinking:
- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

Developing Thinking

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This most effectively occurs through student engagement in inquiry and problem solving when students are challenged to think critically and creatively.

Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "What would happen if …", "Could we find …", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accept responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

K-12 Goals for Developing Identity and Interdependence:
- *understanding, valuing, and caring for oneself*
- *understanding, valuing, and caring for others*
- *understanding and valuing social and environmental interdependence and sustainability.*

Developing Literacies

Through their mathematics learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language is to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. The study of mathematics should encourage the appropriate use of technology. Moreover, students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, oral, written, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings. All too often, it is assumed that symbolic representation is the

K-12 Goals for Developing Literacies:
- *developing knowledge related to various literacies*
- *exploring and interpreting the world through various literacies*
- *expressing understanding and communicating meaning using various literacies.*

only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

> *Students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to a classmate or teacher, or when they formulate a question about something that is puzzling to them. Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols. Misconceptions can be identified and addressed. A side benefit is that it reminds students that they share responsibility with the teacher for the learning that occurs in the lesson.*
>
> *(NCTM, 2000, pp. 60 – 61)*

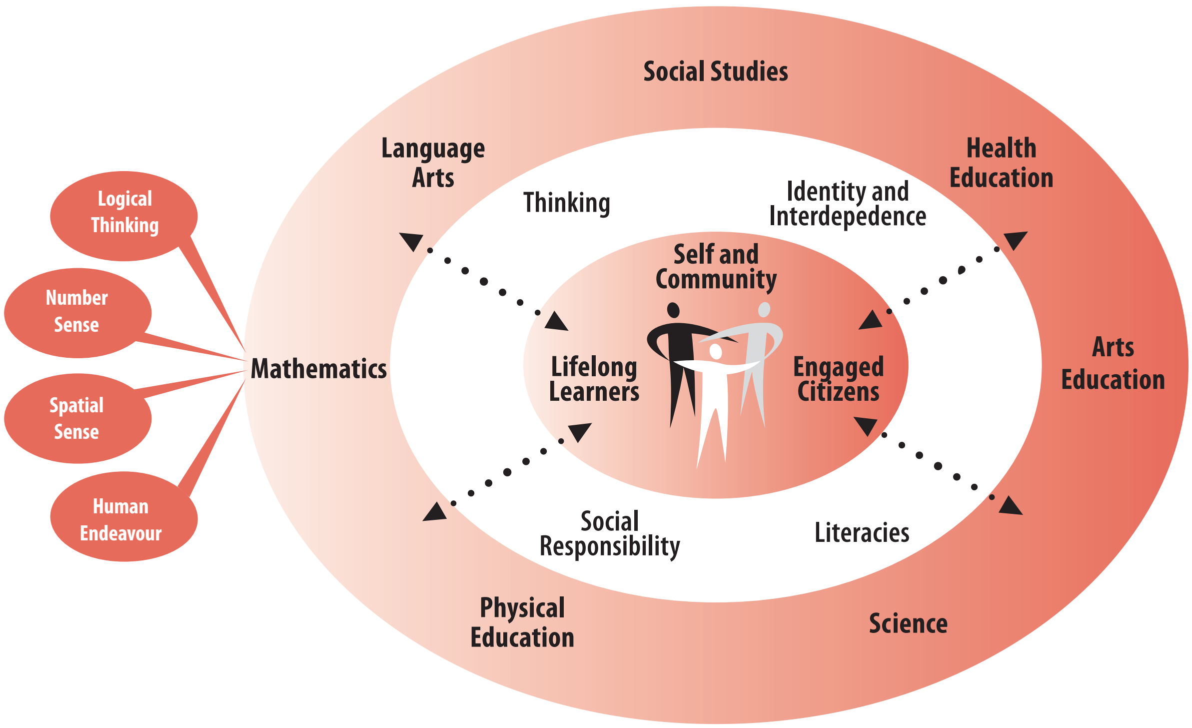
Developing Social Responsibility

K-12 Goals for Developing Social Responsibility:
- *using moral reasoning*
- *engaging in communitarian thinking and dialogue*
- *taking social action.*

As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment be co-constructed by the teacher and students to support respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for situational validity, and strengthened.

Aim and Goals of K-12 Mathematics

The aim of Saskatchewan's K-12 mathematics program is to help students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities and ongoing learnings and work experiences. The mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.



Defined below are four goals for K-12 mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must, therefore, also promote student achievement with respect to the goals.

Logical Thinking

Through their learning of K-12 mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observation
- inductive and deductive thinking

A … feature of the social culture of [mathematics] classrooms is the recognition that the authority of reasonability and correctness lies in the logic and structure of the subject, rather than in the social status of the participants. The persuasiveness of an explanation, or the correctness of a solution depends on the mathematical sense it makes, not on the popularity of the presenter.

(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, Human, 1997, p. 10)

- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and symbolic representations)
- conjecturing and asking "what if" (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

As Grade 3 students are learning about fractions, what are some of the key understandings that students should be attaining? Why? How?

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (including concrete, oral, physical, pictorial, and symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Students also develop understanding of place value through the strategies they invent to compute.
(NCTM, 2000, p. 82)

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

2-D shapes are abstract ideas because they only really exist as parts of 3-D objects. How can you assess to see if your students really understand this connection?

Development of a strong spatial sense requires students to have ongoing experiences with:

- constructing and deconstructing of 2-D shapes and 3-D objects
- investigating and generalizing about relationships between 2-D shapes and 3-D objects
- exploring and abstracting about how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- exploring and generalizing about the movement of 2-D shapes and 3-D objects
- exploring and generalizing about the dimensions of 2-D shapes and 3-D objects
- exploring, generalizing, and abstracting about different forms of measurement and their meaning.

As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop.
(NCTM, 2000, p. 165)

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation of conjectures based upon patterns that are discovered and tested should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

Mathematics as a Human Endeavour

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics

What types of instructional strategies support student attainment of the K-12 mathematics goals?

How can student attainment of these goals be assessed and the results be reported?

- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematically and personally)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Meaning does not reside in tools; it is constructed by students as they use tools.
(Hiebert et al., 1997, p. 10)

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet their particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

The content found within the grade level outcomes for the K-12 mathematics program, and its applications, is first and foremost the vehicle through which students can achieve the four goals of K-12 mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]." This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

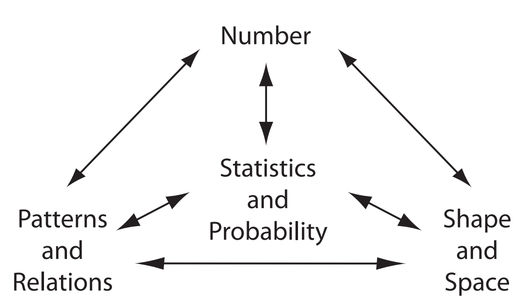
Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from covering to supporting students in discovering mathematical concepts. These facets include:

- organization of the outcomes into strands
- seven mathematical processes
- the difference between covering and discovering mathematics
- development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiqueing statements
- continuum of understanding from concrete to abstract
- modelling and making connections
- role of homework
- importance of ongoing feedback and reflection.

Strands

The content of K-12 mathematics can be organized in a variety of ways. In this curriculum, the outcomes and indicators are grouped according to four strands: **Number, Patterns and Relations, Shape and Space, and Statistics and Probability.** Although this organization implies a relatedness among the outcomes identified in each of the strands, it should be noted the mathematical concepts are interrelated across the strands as well as within strands. Teachers are encouraged to design learning activities that integrate outcomes both within a strand and across the strands so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate strands.



Mathematical Processes

This Grade 3 Mathematics curriculum includes the seven processes identified by the WNCP as being inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

Bracketed letters following each outcome indicate those processes that are most important in the students' learning of the outcome. Teachers should carefully plan to make use of those processes indicated in supporting student achievement of the outcomes.

Communication [C]

Communication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics.
(Hiebert et al., 1997, p. 6)

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, and prior knowledge, the formal language and symbols of mathematics, and new learnings.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding for that terminology.

Concrete, pictorial, symbolic, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.
(Caine & Caine, 1991, p.5)

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

What words would you expect to hear your Grade 3 students saying when they are estimating or using mental mathematics strategies?

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you …?", "Can you …?", or "What if …?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

Mathematical problem-solving often involves moving backwards and forwards between numerical/algebraic representations and pictorial representations of the problem. (Haylock & Cockburn, 2003, p. 203)

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. High-order inquiry challenges students to think and develop a sense of wonder about mathematics.

As an example of reasoning, a Grade 3 student should develop and explain personal strategies for determining perimeter.

> *Posing conjectures and trying to justify them is an expected part of students' mathematical activity.*
> *(NCTM, 2000, p. 191)*

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Visualization [V]

> *[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.*
> *(Armstrong, 1993, p.10)*

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes including aspects such as dimensions and measurements.

Visualization is also important in the students' development of abstraction and abstract thinking and reasoning. Visualization provides a connection between the concrete, physical, and pictorial to the abstract symbolic. Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations as well as the use of communication to develop connections among different contexts, content, and representations.

Technology [T]

> *In Grade 3, there are no outcomes that explicitly require students use of technology. However, are there places where students might use technology to develop their understandings in Grade 3?*

Technology tools contribute to student achievement of a wide range of mathematical outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties

- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Technology should not be used as a replacement for basic understandings and intuition.
(NCTM, 2000, p. 25)

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what needs to be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies and procedures, rules, and problem solving. Any learning in mathematics that is a result of the logical structure of mathematics can and should be constructed by students.

For example, in Grade 3, students should discover the properties of what makes a quantity a fraction, but the teacher needs to cover that such quantities are called fractions.

For example, in Grade 3, the students encounter multiplication for the first time in outcome N3.3 :

N3.3 Demonstrate understanding of multiplication to 5×5 and the corresponding division statements including:

- representing and explaining using repeated addition or subtraction, equal grouping, and arrays
- creating and solving situational questions
- modelling processes using concrete, physical, and visual representations, and recording the process symbolically
- relating multiplication and division.

[C, CN, PS, R]

> *The plus sign, for example, and the symbols for subtraction, multiplication, and division are all arbitrary convention. … Learning most of mathematics, however, relies on understanding its logical structures. The source of logical understanding is internal, requiring a child to process information, make sense of it, and figure out how to apply it.*
> *(Burns & Silbey, 2000, p. 19)*

In this outcome, the term "multiplication" and the symbol "x" are both social conventions of the mathematics the students are learning and, as such, both are something that the teacher must tell the student. Representing and computing multiplication and connecting it to different types of contexts is the mathematics that students need to construct for themselves. This type of learning requires students to work concretely, physically, orally, pictorially, in writing, and symbolically. It also requires that students share their ideas with their classmates and reflect upon how the ideas and understandings of others relate to, inform, and clarify what students individually understand. In this type of learning, the teacher does not tell the students how to do the mathematics but, rather, invites the students to explore and develop an understanding of the logical structures inherent in the mathematics of multiplication. Thus, the teacher's role is to create inviting and rich inquiring tasks and to use questioning to effectively probe and further students' learning.

Development of Mathematical Terminology

> *Teachers should model appropriate conventional vocabulary.*
> *(NCTM, 2000, p. 131)*

Part of learning mathematics is learning how to communicate mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that students already know or that make sense to them.

For example, in outcome SS3.2:

> Demonstrate understanding of measuring mass in g and kg by:
>
> - selecting and justifying referents for g and kg
> - modelling and describing the relationship between the g and kg
> - estimating mass using referents
> - measuring and recording mass.
>
> [C, CN, ME, R]

the terminology, at least in a mathematical sense, of "mass" will likely be new to most of the students. In fact, the new term of "mass" is one that may cause much confusion for students because whether it be at home, in the doctor's office, or when flying somewhere, students are more likely to be familiar with the term “weight”. Although scientifically different in meaning, the general public tends to use the terms “weight” and “mass” interchangeably. It is important that students be introduced to the term “mass” and understand that it is associated with grams and kilograms, versus “weight” which is associated with pounds. First, however, students should be physically, concretely, and orally exploring and describing their understandings of mass, referents for mass, and recording of mass by using the terminology that they know, “weight”. Once students understand the concept, then the term mass should be introduced to the students. It is definitely possible that students may already know the term mass and, in such cases, they should be encouraged to use that term while others in the class may still be referring to weight.

In helping students develop their working mathematical language, it is also important for the teacher to recognize that for many students, including First Nations and Métis, they may not recognize a specific term or procedure, but the student may in fact have a deep understanding of the mathematical topic. Many perceived learning difficulties in mathematics are the result of students' cultural and personal ways of knowing not being connected to formal mathematical language.

In addition, the English language often allows for multiple interpretations of the same sentence, depending upon where the emphasis is placed. For example, consider the sentence “The shooting of the hunters was terrible” (Paulos, 1980, p. 65). Were the hunters that bad of a shot, was it terrible that the hunters got shot, was it terrible that they were shooting, or is this all about the photographs that were taken of the hunters? It is important that students be engaged in dialogue through which they explore possible meanings and interpretations of mathematical statements and problems.

First Nations and Métis Learners and Mathematics

It is important for teachers to realize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understandings. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught to their schema, cultural and environmental content, or real life experiences. A first step in actualization of mathematics from First Nations and Métis perspectives is to empower teachers to understand that mathematics is not acultural. As a result, teachers then realize that the traditional ways of teaching the mathematics are also culturally-biased. These understandings will support the teacher in developing First Nations and Métis students' personal mathematical understandings and mathematical self-confidence and ability through a more holistic and constructivist approach to learning. Teachers need to consider factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

It is important for teachers to recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others, as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding and, as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

For some First Nations and Métis students, the word "equal" may carry the cultural understanding of being "for the good of the community". For example, "equal" sharing of the meat from a hunt may not mean that everyone gets the same amount.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas between cultural groups cannot be assumed to be a direct link. Consider, for example, the concept of "equal", which is a key understanding in this curriculum. The Western understanding of "equal" is 'the same'. In many First Nations and Métis communities, however, "equal" is understood as meaning 'for the good of the community'. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematic classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. Constructivism, inquiry learning, and ethnomathematics allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. Ethnomathematics also shows the relationship between mathematics and cultural anthropology. It is used to translate earlier forms of thinking into modern-day understandings. Individually, and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to them personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthen the learning experiences for all.

Critiquing Statements

One way to assess students' depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. By having students critique such statements, the teacher is able to identify strengths and deficiencies in their understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy. For example, for outcome SS3.4, one of the indicators reads:

Critique the statement "the face of a 3-D object is always a 2-D shape".

The purpose of this indicator is for teachers to assess whether students are able to distinguish between 2-D and 3-D, and how the two terms are being understood. In critiquing this statement, students should be exploring different 3-D objects and using those explorations to justify their responses. The cylinder is an example of a 3-D object that would provide teachers' insights into students' understanding of 2-D and 3-D. Do the students recognize that while the top and bottom of a cylinder are 2-D, the side of the cylinder is actually 3-D? Conversely, if the student identifies a cylinder as being made up of three 2-D shapes, are they wrong? The answer depends on how they identify that third shape. Does the student justify this by visualizing or constructing a rectangle resulting from cutting the side of the cylinder as one would in creating a net?

Critiquing statements is an effective way to assess students individually or as a small or large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important that, in learning mathematics, students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding

It is important for students to use representations that are meaningful to them.
(NCTM, 2000, p. 140)

develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a concrete starting point. Therefore, the teacher must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. For example, when considering a problem about the total number of pencils, some students might find it more concrete to use pictures of pencils as a means of representing the situation. Other students might find coins more concrete because they directly associate money with the purchasing or having of a pencil.

As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following situational question involving subtraction:

> Roger's mother placed 12 apples in a bowl on the centre of the table. The next day, Roger counted only 8 apples in the bowl. How many apples had been taken out of the bowl?

Depending upon how the question is expected to be solved (or if there is any specific expectation), this question can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures, role play), or both.

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

> *A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.*
> *(NCTM, 2000, pp. 139)*

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and symbolic). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you write what you've done here using mathematical symbols?) and by having students compare their representations with others in the class.

In making these connections, students should be asked to reflect upon the mathematical ideas and concepts that are being used in their new models (e.g., I know that addition means to put things together into a group, so I'm going to move the two sets of blocks together to determine the sum).

Making connections also involves looking for patterns. For example, in outcome N3.4:

> Demonstrate understanding of fractions concretely, pictorially, physically, and orally including:
> - representing
> - observing and describing situations
> - comparing
> - relating to quantity.
>
> [C, CN, R]

To develop a deep and meaningful understanding of the students' recognition of patterns, such as the comparison of the size of fractional quantities with a numerator of one and different denominators using different representations, helps the students to develop a stronger understanding of fractions and fractional quantity.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that help students to consolidate new learnings with prior knowledge, explore possible solutions, and apply prior knowledge to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of the drill will vary among different learners. In addition, when used as homework, drill and practice frequently serves to cause frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding of Grade 3 Mathematics, consider outcome SP3.1:

> Demonstrate understanding of first-hand data, using tally marks, charts, lists, bar graphs, and line plots (abstract pictographs), including:
> - collecting, organizing, and representing
> - solving situational questions.
>
> [C, CN, PS, R, V]

As a homework task, students might be asked to collect data over a weekend on a topic of their interest and for the next class

Characteristics of Good Math Homework

- *It is accessible to children at many levels.*
- *It is interesting both to children and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to and types of math homework offered to children throughout the year.*

(Raphel, 2000, p. 75)

be prepared to present how they recorded and organized the data. In the next class, students could share their presentations and then discuss the similarities and differences between the ways that the data were collected and organized. The class could then discuss questions such as the following:

- Which sets of data are easier to understand and why?
- How are the collection and organization methods used by different students the same/different?
- What might be other ways that the data could have been organized?

The teacher can then use the students' prior knowledge and understandings presented to the class to move the students forward in learning. For example, if a student presented a pictograph or concrete graph, the teacher could lead the class forward to line plots by asking "could we use a star instead of the picture or the object?" and "what would the graph look like in that case?".

Ongoing Feedback and Reflection

Feedback can take many different forms. Instead of saying, "This is what you did wrong," or "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about …?"
(Stiff, 2001, p. 70)

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information in the teacher's planning for further and future learnings.

Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.
(NCTM, 2000, p. 72)

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, the addition sign (+) is something which the teacher must introduce and ensure that students know. It is the symbol used to show the combination or addition of two quantities. The process of adding, however, and the development of addition and subtraction facts should be discovered through the students' investigation of patterns, relationships, abstractions, and generalizations.

It is important for teachers to analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to consider opportunities they can provide for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences… A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.
(Haylock & Cockburn, 2003, p. 18)

It is important that a mathematics learning environment include effective interplay of:

- reflection and metacognition
- exploration of patterns and relationships
- sharing of ideas and problems
- consideration of different perspectives
- decision making
- generalization and abstraction
- verifying and proving
- modeling and representing
- making connections.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

What types of things might you hear or see in a Grade 3 classroom that would indicate to you that students were learning for deep understanding?

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

> *Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.*
> *(Mills & Donnelly, 2001, p. xviii)*

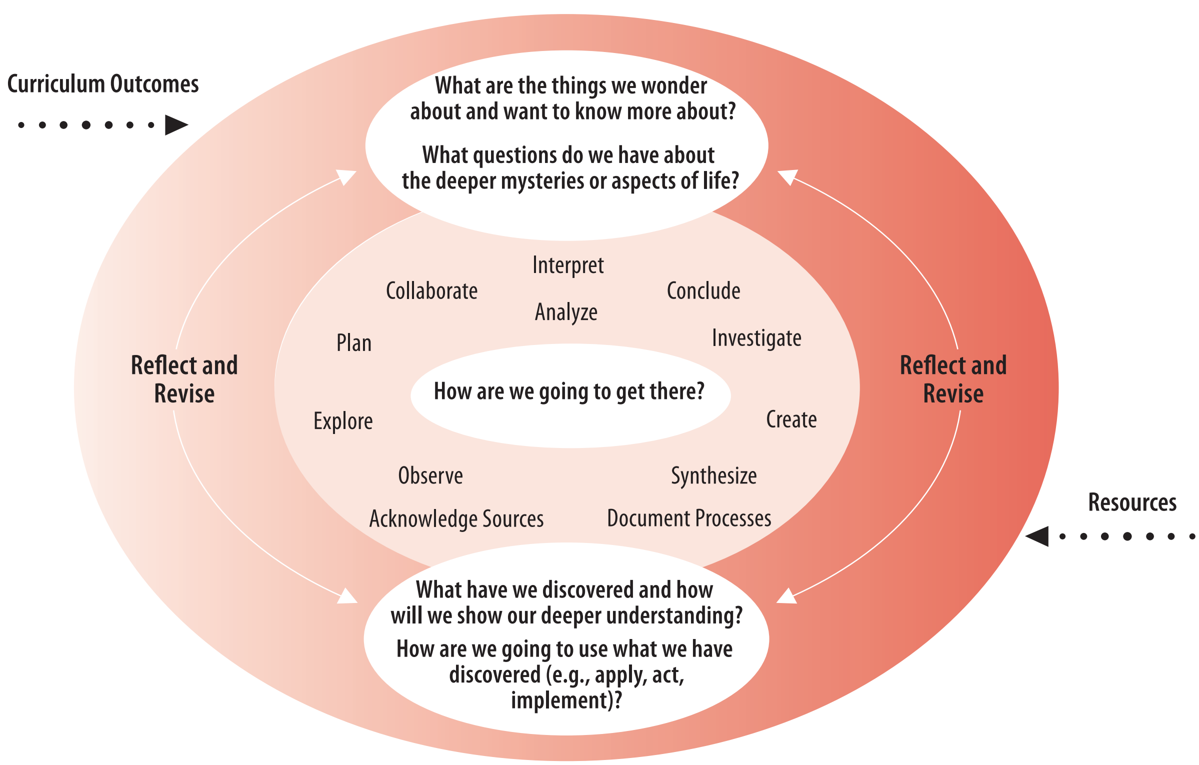
Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are directly involved and engaged in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning and mastery of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with various phases of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge. The following graphic shows the cyclical inquiry process.

Constructing Understanding Through Inquiry



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, conclude, document, reflect on learning, and develop new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students too must understand this difference.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Questions may be one of the most powerful technologies invented by humans. Even though they require no batteries and need not be plugged into the wall, they are tools which help us make up our minds, solve problems, and make decisions.
(Jamie McKenzie, in Schuster & Canavan Anderson, 2005, p. 1)

Effective questions:
- *cause genuine and relevant inquiry into the important ideas and core content.*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions.*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answers.*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons.*
- *spark meaningful connections with prior learning and personal experiences.*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:
- "When would you want to add two numbers less than 100?"
- "How do you know you have an answer?"
- "Will this work with every number? Every similar situation?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning in mathematics. Questioning should also be used to encourage students to reflect on the inquiry process and the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- *help students make sense of the mathematics.*
- *are open-ended, whether in answer or approach. There may be multiple answers or multiple approaches.*
- *empower students to unravel their misconceptions.*
- *not only require the application of facts and procedures but encourage students to make connections and generalizations.*
- *are accessible to all students in their language and offer an entry point for all students.*
- *lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.*

(Schuster & Canavan Anderson, 2005, p. 3)

As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.
(Schuster & Canavan Anderson, 2005, p. 1)

Reflection and Documentation of Inquiry

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, drafts, models, and works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

It is important that students are required to engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

Outcomes and Indicators

Number

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes (What students are expected to know and be able to do.)	**Indicators** (Students who have achieved this outcome should be able to:)
N3.1 Demonstrate understanding of whole numbers to 1000 (concretely, pictorially, physically, orally, in writing, and symbolically) including: • ***representing (including place value)*** • ***describing*** • ***estimating with referents*** • ***comparing two numbers*** • ***ordering three or more numbers.*** ***[C, R, V]***	a. Observe, represent, and state the sequence of numbers for a given skip counting pattern (forwards or backwards) including: • by 5s, 10s, or 100s using any starting point • by 3s, 4s, or 25s using starting points that are multiples of 3, 4, and 25 respectively. b. Analyze a sequence of numbers to identify the skip counting pattern (forwards or backwards) including: • by 5s, 10s, or 100s using any starting point • by 3s, 4s, or 25s using starting points that are multiples of 3, 4, and 25 respectively. c. Create and explain the reasoning for a sequence of numbers that have different skip counting patterns in it (e.g., 3, 6, 9, 12, 16, 20, 24). d. Explore and present First Nations and Métis methods of determining and representing whole number quantities (e.g., in early Cree language, quantity was a holistic concept addressing sufficiency for a group such as none/nothing, a little bit/not many, and a lot). e. Analyze a proposed skip counting sequence for errors (including omissions and incorrect values) and explain the errors made. f. Solve situational questions involving the value of coins or bills and explain the strategies used (such as grouping or skip counting). g. Identify errors (such as the use of commas or the word 'and') made in speech or in the writing of quantities that occur in conversations (personal), recordings (such as TV, radio, or podcasts) and written materials (such as the Internet, billboards, or newspapers).

Outcomes	Indicators
N3.1 *(continued)*	h. Write (in numerals for all quantities, and in words if the quantity is a multiple of 10 and less than 100 or a multiple of 100 and less than 1000) and read aloud statements relevant to one's self, family, or community that contain quantities up to 1000 (e.g., a student might write, "Our town has a population of 852" and read the numeral as eight hundred fifty-two). i. Create different decompositions of the same quantity (concretely using proportional or non-proportional materials, physically, orally, or pictorially), explain how the decompositions represent the same overall amount, and record the decompositions as symbolic expressions (e.g., 300 – 44 and 236 + 20 are two possible decompositions that could be given for 256). j. Sort a set of numbers into ascending or descending order and justify the result (e.g., using hundred charts, a number line, or by explaining the place value of the digits in the numbers). k. Create as many different 3-digit numerals as possible, given three non-repeating digits, and sort the numbers in ascending or descending order. l. Select and use referents for 10 or 100 to estimate the number of groups of 10 or 100 in a set of objects. m. Analyze a sequence of numbers and justify the conclusion of whether or not the sequence is ordered. n. Identify missing whole numbers on a section of a number line or within a hundred chart. o. Record, in more than one way, the quantity represented by proportional (e.g., base ten blocks) or non-proportional (e.g., coins) concrete materials. p. Explain, using concrete materials or pictures, the meaning of each digit in a given 3-digit numeral with all the same digits. q. Provide examples of how different representations of quantities, including place value, can be used to determine sums and differences of whole numbers.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N3.2 Demonstrate understanding of addition of whole numbers with answers to 1000 and their corresponding subtractions (limited to 1, 2, and 3-digit numerals) including: • ***representing strategies for adding and subtracting concretely, pictorially, and symbolically*** • ***solving situational questions involving addition and subtraction*** • ***estimating using personal strategies for adding and subtracting.*** ***[CN, ME, PS, R, V]***	a. Describe personal mental mathematics strategies that could be used to determine a given basic fact, such as: • doubles (e.g., for 6 + 8, think 7 + 7) • doubles plus one (e.g., for 6 + 7, think 6 + 6 + 1) • doubles take away one (e.g., for 6 + 7, think 7 + 7 – 1) • doubles plus two (e.g., for 6 + 8, think 6 + 6 + 2) • doubles take away two (e.g., for 6 + 8, think 8 + 8 – 2) • making 10 (e.g., for 6 + 8, think 6 + 4 + 4 or 8 + 2 + 4) • commutative property (e.g., for 3 + 9, think 9 + 3) • addition to subtraction (e.g., for 13 – 7, think 7 + ? = 13) b. Observe and generalize personal strategies from different types of representations for adding 2-digit quantities (given concrete materials, pictures, and symbolic decompositions) such as: • Adding from left to right (e.g., for 23 + 46 think 20 + 40 and 3 + 6) • Taking one or both addends to the nearest multiple of 5 or 10 (e.g., for 28 + 47, think 30 + 47 - 2, 50 + 28 – 3, or 30 + 50 – 2 – 3) • Using doubles (e.g., for 24 + 26, think 25 + 25, or for 25 + 26, think 25 + 25 + 1). c. Observe and generalize personal strategies for subtracting 2-digit quantities (given concrete materials, pictures, and symbolic decompositions) such as: • Taking the subtrahends to the nearest multiple or 10 (e.g., for 48 - 19, think 48 – 20 + 1) • Thinking of addition (e.g., 62 – 45, think 45 + 5, 50 + 12 to get from 45 to 62, so the difference is 5 + 12) • Using doubles (e.g., for 25 - 12, think 12 + 12 = 24 and 24 is one less than 25, so difference is 12 + 1). d. Apply and explain personal mental mathematics strategies to determine the sums and differences of two-digit quantities. e. Create a situational question that involves either addition or subtraction and that has a given quantity as the solution. f. Model (concretely or pictorially) a process for the addition of two or more given quantities (with a sum less than 1000) and record the process symbolically. g. Model (concretely or pictorially) a process for the subtraction of two or more quantities (less than 1000) and record the process symbolically.

Outcomes	**Indicators**
N3.2 *(continued)*	h. Generalize (orally, in writing, concretely, or pictorially) personal strategies for estimating the sum or difference of two 2-digit quantities. i. Extend personal mental mathematics strategies to determine sums and differences (of quantities less than 1000) and explain the reasoning used. j. Transfer knowledge of the basic addition facts up to 18 and the related subtraction facts to determine the sums and differences of quantities less than 1000. k. Generalize rules for the addition and subtraction of zero. l. Provide examples to show why knowing about place value is useful when adding and subtracting quantities.

Mathematics 3

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
Note: *The focus of this outcome is for the students to become familiar with multiplication and division and strategies for mentally determining products and quotients. It is not intended that students memorize the basic facts.*	
N3.3 Demonstrate understanding of multiplication to 5 x 5 and the corresponding division statements including: • ***representing and explaining using repeated addition or subtraction, equal grouping, and arrays*** • ***creating and solving situational questions*** • ***modelling processes using concrete, physical, and visual representations, and recording the process symbolically*** • ***relating multiplication and division.*** ***[C, CN, PS, R]***	a. Observe and describe situations relevant to self, family, or community that can be represented by multiplication and write and solve a multiplication statement for each situation. b. Observe and describe situations relevant to self, family, or community that can be represented by equal sharing or grouping and write and solve a division statement for each situation. c. Explain and represent concretely, pictorially, orally, or physically, as well as symbolically, the relationship between repeated addition and multiplication and the relationship between repeated subtraction and division. d. Represent and solve an orally presented multiplication or division statement, concretely, physically, or pictorially, using equal groupings, an array, repeated addition, or repeated subtraction (e.g., 3 x 4 shown using equal groupings of snowballs). e. Apply and explain personal strategies for determining products and quotients. f. Model the commutative property of multiplication and write the symbolic multiplication equation represented. g. Represent and solve an orally presented situational question that involves division. h. Relate multiplication and division orally and by using concrete, physical, or pictorial models, including repeated addition/subtraction and arrays/dimensions. i. Create multiplication or division statements and determine the resulting products or quotients related to a given situational question. j. Create and solve a situational question that relates to a given symbolic multiplication or division statement.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N3.4 Demonstrate understanding of fractions concretely, pictorially, physically, and orally including: • ***representing*** • ***observing and describing situations*** • ***comparing*** • ***relating to quantity.*** *[C, CN, R]*	a. Identify and observe situations relevant to self, family, or community in which fractional quantities would be measured or used and explain what the fraction quantifies. b. Explore First Nations and Métis methods of observing and representing fractional quantities (e.g., consider the concept of sharing from a First Nations or Métis holistic worldview). c. Explain the relationship of a representation of a fraction to both a quantity of zero and a quantity of one (the whole or entire group, region, or length). d. Divide a whole, group, region, or length into equal parts (concretely, physically, or pictorially), demonstrate that the parts are equal in quantity, and name the quantity represented by each part. e. Analyze a set of diagrams or concrete representations to sort the representations into those that represent the same fraction and those that do not, and explain the sorting. f. Analyze representations of a set of fractions of a whole, group, region, or length that all have the same numerator (e.g., 2/3, 2/4, 2/5) and explain what about the fractional quantities is similar and what is different. g. Analyze representations of a set of fractions of a whole, group, region, or length that all have the same denominator (e.g., 0/5, 1/5, 2/5, 3/5, 4/5, 5/5) and explain what about the fractional quantities is similar and what is different. h. Explain the role of the numerator and denominator in a fraction. i. Demonstrate how a fraction can represent a different amount if a different size of whole, group, region, or length is used. j. Compare, concretely, pictorially, physically, or orally, and order a set of fractions with either equivalent denominators or equivalent numerators. k. Represent a fraction as part of a whole, group, region, or length and explain the representation. l. Explain how a region can be divided into unequal parts, but the parts still represent a fraction of the region (e.g., Canada divided into provinces and territories which are not equal in area).

Patterns and Relations

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
	Note: *It is intended that decreasing patterns will not go past zero.*
P3.1 Demonstrate understanding of increasing and decreasing patterns including: • ***observing and describing*** • ***extending*** • ***comparing*** • ***creating patterns using manipulatives, pictures, sounds, and actions.*** ***[C, CN, PS, R, V]***	a. Identify and observe situations relevant to self, family, and community that contain an increasing or decreasing pattern, identify the starting value of the pattern, and describe the rule for the pattern and how the pattern would continue. b. Verify (concretely, visually, orally, pictorially, or physically) whether or not a given sequence of numbers represents an increasing or decreasing pattern. c. Observe various patterns (increasing or decreasing) found on a hundred chart, such as horizontal, vertical, and diagonal patterns, and describe the pattern rule. d. Compare visual patterns for skip counting (forwards or backwards) by 2s, 5s, 10s, 25s, and 100s and relate to increasing and decreasing patterns. e. Visualize and create oral, concrete, physical, pictorial, or symbolic representations for a given increasing or decreasing pattern rule and explain how the representations are related. f. Create a concrete, physical, pictorial, or symbolic pattern (increasing or decreasing) and describe the pattern rule. g. Describe strategies used to solve situational questions involving increasing or decreasing patterns, including determining missing elements within the pattern. h. Research (e.g., through Elders, traditional knowledge keepers, naturalists, and media) and present about the role and significance of increasing and decreasing patterns (e.g., making of a star blanket, beading, music, and patterns found in nature) in First Nations and Métis practices, lifestyles, and worldviews.

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P3.2 Demonstrate understanding of equality by solving one-step addition and subtraction equations involving symbols representing an unknown quantity. [C, CN, ME, R]	a. Share, compare, and distinguish between understandings and uses of the word equal, including those represented in First Nations and Métis worldviews. b. Observe and describe situations relevant to self, family, or community in which a symbol could be used to represent an unknown quantity. c. Explain the purpose of the symbol, such as a triangle or a circle, in an addition or subtraction equation. d. Compare two equations involving the same operations and quantities, but using different symbols. e. Solve addition and subtraction equations concretely, pictorially, or physically. f. Verify (concretely, pictorially, or physically) which of a set of given quantities is the solution to a one-step addition or subtraction equation and explain the reasoning. g. Generalize strategies, including guess and test, for solving one-step addition and subtraction equations and verify the strategies concretely, pictorially, or physically. h. Explain why the unknown in a given addition or subtraction equation has only one value. i. Create and solve one-step equations related to situational questions. j. Create and solve situational questions that relate to given one-step equations.

Shape and Space

Goals: Spatial Sense, Logical Thinking, Number Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
SS3.1 Demonstrate understanding of the passage of time including: • ***relating common activities to standard and non-standard units*** • ***describing relationships between units*** • ***solving situational questions.*** ***[C, CN, PS, R]***	a. Observe and describe activities relevant to self, family, and community that would involve the measurement of time. b. Explore the meaning and use of time-keeping language from different cultures, including First Nations and Métis. c. Select and use a personally relevant non-standard unit of measure for the passage of time (such as television shows, a pendulum swing, sunrise, sundown, moon cycles, and hunger patterns) and explain the choice. d. Suggest and sort activities into those that can or cannot be accomplished in a minute, hour, day, month, or year. e. Select and justify personal referents for minutes and hours. f. Create and solve situational questions using the relationship between the number of minutes in an hour, days in a particular month, days in a week, hours in a day, weeks in a year, or months in a year (e.g., "A student was on holiday for 10 days. Is that more or less than one week long?"). g. Identify the day of the week, the month, and the year for an indicated date on a calendar. h. Identify today's date, and then explain how to determine yesterday's and tomorrow's date. i. Locate a stated or written date (day, month, and year) on a calendar and explain the strategy used. j. Identify errors in the ordering of the days of the week and the months of the year. k. Create a calendar using the days of the week, the calendar dates, and personally relevant events. l. Describe ways in which the measurement of time is cyclical.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
SS3.2 Demonstrate understanding of measuring mass in g and kg by: • ***selecting and justifying referents for g and kg*** • ***modelling and describing the relationship between g and kg*** • ***estimating mass using referents*** • ***measuring and recording mass.*** ***[C, CN, ME, R]***	a. Observe and describe situations relevant to self, family, and community that involve measuring mass. b. Create and solve situational questions that involve the estimating or measuring of mass using g or kg. c. Analyze 3-D objects to determine personal referents for 1 kg, 100 g, 10 g, and 1 g. d. Analyze the relationships between 1 g, 10 g, 100 g, 1000 g, and 1 kg and explain the strategies used (e.g., 1 kg is heavier than 100 g, 10 g, and 1 g, or 1 kg is the same mass as 1000 g.) e. Select, with justification, an appropriate unit for measuring the mass of a given 3-D objects (e.g., kg would be used to measure a motorbike). f. Determine, using a scale, and record the mass of an object relevant to one's self, family, or community. g. Estimate the mass of an object relevant to one's self, family, or community and explain the strategy used. h. Directly compare the mass of two 3-D objects and then verify the comparison by measuring the actual masses using a scale. i. Generalize statements about the mass of a specific amount of matter when reformed into different shapes or sizes (e.g, use clay to make an object, measure the mass of the object, reform the clay into another object and measure the mass of the two objects; an empty balloon versus a full balloon; or water versus ice). j. Observe and document conversations, mass media reports, and other forms of text that use the term "weight" rather than "mass".

Mathematics 3

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS3.3 Demonstrate understanding of linear measurement (cm and m) including: • ***selecting and justifying referents*** • ***generalizing the relationship between cm and m*** • ***estimating length and perimeter using referents*** • ***measuring and recording length, width, height, and perimeter.*** ***[C, CN, ME, PS, R, V]***	a. Observe and describe situations relevant to self, family, and community that involve measuring lengths, including perimeter, in cm or m. b. Measure and compare different lengths on 3-D objects to select personally relevant referents for 1 cm, 10 cm, and 1 m. c. Create models to generalize a numerical relationship between cm and m (i.e., 100 cm is equivalent to 1 metre). d. Pose and solve situational questions that involve the estimating or measuring of length (including perimeter) using cm or m. e. Identify and determine the length of the dimensions of a personally relevant 2-D shape or 3-D object. f. Explain why sometimes different names are used for different length measurements (e.g., height, width, or depth). g. Sketch a line segment of an estimated length and describe the strategy used. h. Draw a line segment of a given length and explain the process used. i. Relate measuring using a referent for 10 cm to skip counting quantities by 10s. j. Create a picture of a 2-D shape with specified length and width (or length and height) and explain whether the 2-D shape was constructed using estimates or actual lengths. k. Measure and record the perimeter of regular 2-D polygons and circles located on 3-D objects, and explain the strategy used. l. Measure and record the perimeter of a given irregular 2-D shape, and explain the strategy used. m. Construct or draw more than one 2-D shape for the same given perimeter (cm, m). n. Estimate the perimeter of a given 2-D shape (cm, m) using personal referents and explain the strategies used. o. Critique the statement "perimeter is a linear measurement". p. Sort a set of 2-D shapes into groups with equal perimeters.

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS3.4 Demonstrate understanding of 3-D objects by analyzing characteristics including faces, edges, and vertices. ***[C, V]***	a. Observe and describe the faces, edges, and vertices of given 3-D objects, including cubes, spheres, cones, cylinders, pyramids, and prisms (e.g., drum, tipi, South American Pyramids, and other objects from the natural environment). b. Critique the statement "the face of a 3-D object is always a 2-D shape". c. Observe and describe the 2-D shapes found on a 3-D object. d. Construct a skeleton of a given 3-D object and describe how the skeleton relates to the 3-D object. e. Determine the number of faces, edges, and vertices of a given 3-D object and explain the reasoning and strategies. f. Critique the statement "a vertex is where three faces meet". g. Sort a set of 3-D objects according to the faces, edges, or vertices and explain the sorting rule used.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS3.5 Demonstrate understanding of 2-D shapes (regular and irregular) including triangles, quadrilaterals, pentagons, hexagons, and octagons including: • ***describing*** • ***comparing*** • ***sorting.*** ***[C, CN, R]***	a. Identify the sorting rule used on a pre-sorted set of polygons. b. Generalize definitions for regular and irregular polygons based on a concept attainment activity or from pre-sorted sets. c. Observe, describe the characteristics of, and sort polygons found in situations relevant to self, family, or community (including First Nations and Métis), into irregular and regular polygons (e.g., the bottom of a kamatiq, the screen of a TV, the bottom of a curling broom, and an arrowhead). d. Analyze irregular and regular polygons in different orientations in terms of the characteristics of the polygons (such as number or measurement of sides and angles).

Statistics and Probability	
Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour	
Outcomes	**Indicators**
SP3.1 Demonstrate understanding of first-hand data using tally marks, charts, lists, bar graphs, and line plots (abstract pictographs), through: • ***collecting, organizing, and representing*** • ***solving situational questions.*** ***[C, CN, PS, R, V]***	a. Observe and describe situations relevant to self, family, or community in which a particular type of data recording or organizing strategy might be used, including tally marks, charts, lists, and knots on a sash. b. Analyze a set of line plots to determine the common attributes of line plots. c. Create a line plot from a pictograph. d. Analyze a set of bar graphs to determine the common attributes of bar graphs. e. Answer questions related to the data presented in a bar graph or line plots. f. Collect and represent data using bar graphs or line plots. g. Pose and solve situational questions related to self, family, or community by collecting and organizing data, representing the data using a bar graph or line plot, and interpreting the data display. h. Analyze interpretations of bar graphs or line plots and explain whether or not the interpretation is valid based on the data display. i. Examine how various cultures past and present, including First Nations and Métis, collect, represent, and use first-hand data.

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- ☑ Achievement of provincial curriculum outcomes
- ☑ Effectiveness of teaching strategies employed
- ☑ Student self-reflection on learning.

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Assembling evidence from a variety of sources is more likely to yield an accurate picture.
(NCTM, 2000, p. 24)

Reporting of student achievement must be in relation to curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Grade 3 Mathematics. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

Assessment should not merely be done to students; rather it should be done for students.
(NCTM, 2000, p. 22)

Assessment as learning involves student reflection on and monitoring of her/his own progress related to curricular outcomes and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

What are examples of assessments as learning that could be used in Grade 3 Mathematics and what would be the purpose of those assessments?

> *Assessment should become a routine part of the ongoing classroom activity rather than an interruption.*
> *(NCTM, 2000, p. 23)*

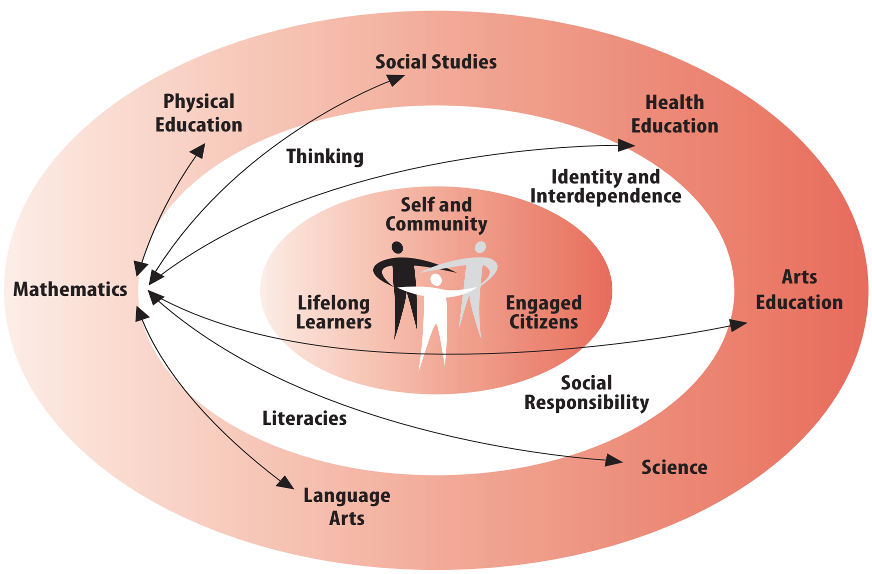
Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

In mathematics, students need to be regularly engaged in assessment as learning. The assessments used should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the mathematical content for Grade 3.

Connections with Other Areas of Study

There are many possibilities for connecting Grade 3 mathematical learning with the learning occurring in other subject areas. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. Following are just a few of the ways in which mathematics can be connected to other subject areas (and other subject areas connected to mathematics) at Grade 3.



Arts Education – In Grade 3, very strong connections exist between the students' study of arts education and mathematics, especially in relation to Outcome P3.1:

> Demonstrate understanding of increasing and decreasing patterns including:
> - observing and describing
> - extending
> - comparing
> - creating patterns using manipulatives, pictures, sounds, and actions.
>
> [C, CN, PS, R, V]

Students can use Dance, Drama, Visual Arts, and Music to explore and demonstrate their understandings of increasing and decreasing patterns. Arts education at Grade 3 is an especially rich area for students to explore the decreasing aspects of patterns. Students can explore the use of decreasing dialogue, movement, colours, or sound and discuss how these changes impact the viewer and the creator of the work of art.

English Language Arts (ELA) – ELA and mathematics share a common interest in students developing their abilities to develop, reflect upon, and communicate about their learnings through viewing, listening, reading, representing, speaking, and writing. As an example of how mathematics involves these strands of language, consider outcome P3.1:

> Demonstrate understanding of increasing and decreasing patterns including:
> - observing and describing
> - extending
> - comparing
> - creating patterns using manipulatives, pictures, sounds, and actions.
>
> [C, CN, PS, R, V]

In order for students to achieve this outcome, their learning experiences need to engage them in the analysis of, comparison of, and construction of increasing and decreasing patterns. Students need to listen to and view a wide variety of representations of increasing and decreasing patterns in order to recognize and describe the characteristics of such patterns. As the students experience different increasing and decreasing patterns through viewing and listening, they learn to describe orally and in writing (including symbolically) the relationships between different representations of the same and different patterns. When students share their communications about different representations and different patterns, students again are engaged in viewing and listening, along with reading. Finally, students use all of their understandings about increasing and decreasing patterns to create new representations for a given pattern, or to create and describe a pattern of their own design.

It is important in the mathematics classroom to engage students in communication of all forms, providing students opportunities to work within the contexts that they are most comfortable with, while continuing to challenge the students to improve all aspects of their communication skills.

Health Education – Grade 3 mathematics and health education can be connected in student learning by engaging the students in data collection and interpretation found within outcome SP3.1.

> Demonstrate understanding of first-hand data using tally marks, charts, lists, bar graphs, and line plots (abstract pictographs), including:
> - collecting, organizing, and representing
> - solving situational questions.
>
> [C, CN, PS, R, V]

Students could analyze the labels on different foods by creating bar graphs for each type of food and comparing the resulting graphs. Similarly, the students could use a chart or lists to collect data about different types of drugs and to categorize that data as prescription/non prescription or legal/illegal. As well, students could collect data regarding types of safe behaviours/practices used within their homes, at school, or in other communities that they are part of and discuss and implement ways to prevent, avoid, or reduce those safety risks. Students could continue to collect relevant data throughout the year to report on the effectiveness of the measures they chose to implement.

Physical Education – There are many connections between physical education and mathematics possible at Grade 3. Students can collect and represent data regarding the skills and strategies that they engage in. This data can then be interpreted by the students to answer questions that are relevant to them. As well, students can combine their learning of locomotor skills, non-locomotor skills, and manipulative skills to demonstrate their understanding of increasing and decreasing patterns by creating new representations for a pattern, extending a pattern, comparing patterns, and creating original patterns.

The students' study of locomotor skills can also be used as the context in which students engage with notions of length and perimeter such as found in outcome SS3.3:

> Demonstrate an understanding of linear measurement (cm and m) including:
> - selecting and justifying referents
> - generalizing the relationship between cm and m
> - estimating length and perimeter using referents
> - measuring and recording length, width, height, and perimeter.
>
> [C, CN, ME, PS, R, V]

Students may choose to use a locomotor skill that they have been developing as a referent for m which they could then use to estimate lengths and perimeters. Students could engage in discussions comparing personal referents and why one locomotor action may not be referent for one student while it is for a second student.

Science – There are many direct connections between Grade 3 science and mathematics. In both the Exploring Soils unit and the Plant Growth and Changes unit in science, students are collecting data and creating bar graphs to display these data. These learning experiences also contribute directly to the students' learning towards outcome SP3.1. In addition, the students have opportunities to develop their understanding of increasing and decreasing patterns in both the Plant Growth and Changes unit and the Structures and Materials unit. Students can analyze, represent, extend, and compare the growth of plants as well as the strength and stability of different 3-D objects. Further, the students' study of Structures and Materials in science directly relates to their study of outcome SS3.4:

> Demonstrate understanding of 3-D objects by analyzing characteristics including faces, edges, and vertices.
> [C, V]

As students are constructing replicas of everyday structures, analyzing simple structures for effectiveness, and assessing the efficient use of materials, safety, and appropriateness, the students can be using and deepening their mathematical understandings of the characteristics of 3-D objects.

Social Studies – In Grade 3 social studies, students explore the communities to which they belong. These explorations can easily be tied to the students' mathematical learnings for outcome SP3.1:

> Demonstrate understanding of first-hand data using tally marks, charts, lists, bar graphs, and line plots (abstract pictographs), including:
> - collecting, organizing, and representing
> - solving situational questions.
>
> [C, CN, PS, R, V]

As the students explore different aspects of communities, they can record data they collect using tally marks, charts, and lists. These data can then be represented using bar graphs or line plots, and students can interpret the data to answer personally relevant questions.

Glossary

Addend: Any quantity being added to another quantity (e.g., in the expression 32 + 57, both 32 and 57 are addends).

Array: A visual, concrete, or pictorial pattern arranged in a grid formation. For example, seating in an auditorium could be represented by an array.

Attributes: Characteristics of 2-D shapes and 3-D objects that can be used to compare and sort sets of 2-D shapes and 3-D objects (e.g., colour, relative size, number of corners, number of lines of symmetry).

Bar Graph: A graph in which data are represented by horizontal or vertical bars. Each bar represents the quantity of data of a certain type or category (e.g., red, blue, green, or yellow; 1998, 1999, 2000, 2001; or car, truck, bus, bike, walk).

Benchmarks: Numeric quantities used to compare and order other numeric quantities. For example, 0, 5, 10, and 20 are often used as benchmarks when placing whole numbers on a number line.

Denominator: The bottom number in a fraction that defines how many equal parts are in a whole.

Equality as a Balance and Inequality as Imbalance: The equal sign represents the idea of equivalence. For many students, it means "do the question". For some students, the equal sign in an expression such as 2 + 5 = means to add. When exploring equality and inequality, by using objects on a balance scale, students discover the relationships between and among the mass of the objects. The equal sign in an equation is like a scale: both sides, left and right, must be the same in order for the scale to stay in balance and the equation to be true. When the scale is imbalanced, the equation is not true. Using 2 + 5 = □, rather than simply 2 + 5 = helps students understand that the equal sign (=) represents equality rather than "do the work" or "do the question".

Equation: A mathematical statement that shows that two different expressions are equal in quantity. The two expressions are separated by the equal sign (=).

Ethnomathematics: The study of the relationship between mathematics and culture.

First-hand Data: Data that has been directly collected by the person using it (e.g., questionnaire data).

Fraction: A description of quantity as part of a whole.

Interdisciplinary: Disciplines connected by common concepts and skills embedded in disciplinary outcomes.

Line plot: A stylized version of a pictograph where the symbol * is used in place of one picture.

Minuend: In a subtraction sentence, the quantity that is being decreased (e.g., in the subtraction sentence 84 – 55, 84 is the minuend).

Multidisciplinary: Discipline outcomes organized around a theme and learned through the structure of the disciplines.

Number, Numeral, Digit: A number is the name that we give to quantities. For example, there are seven days in a week, or I have three brothers – both seven and three are numbers in these situations because they are defining a quantity. The symbolic representation of a number, such as 287, is called the numeral. If 287 is not being used to define a quantity, we call it a numeral.

Numerals, as the symbolic representation of numbers, are made up of a series of digits. The Hindu-Arabic number system that we use has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note: sometimes students are confused between these digits and their finger digits – this is because they count their fingers starting at one and get to ten rather than zero to nine.) These digits are also numerals and can be numbers (representing a quantity), but all numbers and all numerals are combinations of digits. The placement of a digit in a number or numeral affects the place value of the digit and, hence, how much of the quantity that it represents. For example, in 326, the 2 is contributing 20 to the total, while in 236 the 2 contributes 200 to the total quantity.

Numerator: The top number in a fraction. The numerator tells how many (the quantity) parts are present or being considered. The number of parts making up the whole is defined by the denominator of the fractions.

Object: Object is used to refer to a three-dimensional geometrical figure. To distinguish this meaning from that of shape, the word "object" is preceded by the descriptor "3-D".

Pattern Rule: A description of how consecutive terms or elements in a pattern are determined.

Personal Strategies: Personal strategies are strategies that the students have constructed and understand. Outcomes and indicators that specify the use of personal strategies convey the message that there is not a single procedure that is correct. Students should be encouraged to explore, share, and make decisions about what strategies to use in different contexts. Development of personal strategies is an indicator of the attainment of deeper understanding.

Pictograph: A graph which uses pictures or symbols to show how often something occurs.

Referents: A concrete approximation of a quantity or unit of measurement. For example, seeing what 25 beans in a container looks like makes it possible to estimate the number of beans the same container will hold when it is full of the same kind of beans. Compensation must be made if the container is filled with smaller or larger beans than the referent or if the size or shape of the container is changed.

Representations: Mathematical ideas can be represented and manipulated in a variety of forms including concrete manipulatives, visual designs, sounds and speech, physical movements, and symbolic notations (such as numerals and operation signs). Students need to have experiences in working with many different types of representations, and in transferring and translating knowledge between the different forms of representations.

Shape: In this curriculum, shape is used to refer to two-dimensional geometric figures and is thus preceded by "2-D". The term shape is sometimes also used in mathematics resources and conversations to refer to three-dimensional geometric figures. It is important that students learn to be clear in identifying whether their use of the term shape is in reference to a 2-D or 3-D geometrical figure.

Subtrahend: In a subtraction statement, the quantity that is being subtracted (e.g., in the subtraction statement 90 – 26, 26 is the subtrahend).

Tally Marks: One way to collect and organize data. Each tally mark (often shown as a downward stroke |) represents one time that value appears in the data . Frequently, tally marks are grouped into sets of fives (four downward strokes and one cross-stroke) for ease of counting.

Transdisciplinary: All knowledge interconnected and interdependent; real-life contexts emphasized and investigated through student questions.

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Feedback Form

The Ministry of Education welcomes your response to this curriculum and invites you to complete and return this feedback form.

Document Title: **Mathematics Grade 3 Curriculum**

1. Please indicate your role in the learning community

☐ parent ☐ teacher ☐ resource teacher

☐ guidance counsellor ☐ school administrator ☐ school board trustee

☐ teacher librarian ☐ school community council member

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What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

☐ print

☐ online

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3. How does this curriculum address the needs of your learning community or organization? Please explain.

4. Please respond to each of the following statements by circling the applicable number.

The curriculum content is:	Strongly Agree	Agree	Disagree	Strongly Disagree
a. appropriate for its intended purpose	1	2	3	4
b. suitable for your learning style (e.g., visuals, graphics, texts)	1	2	3	4
c. clear and well organized	1	2	3	4
d. visually appealing	1	2	3	4
e. informative	1	2	3	4

5. Explain which aspects you found to be:

   Most useful:

   Least useful:

6. Additional comments:

7. Optional:

   Name: ____

   School: ____

   Phone: ____________________ Fax: ____________________

Thank you for taking the time to provide this valuable feedback.

Please return the completed feedback form to:

Executive Director
Curriculum and E-Learning Branch
Ministry of Education
2220 College Avenue
Regina SK S4P 4V9
Fax: 306-787-2223