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Saskatchewan Curriculum

Mathematics

6

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Introduction

As a Required Area of Study, mathematics is to be allocated 210 minutes per week for the entire school year at Grade 6. It is important that students receive the full amount of time allocated to their mathematical learning and that the learning be focused upon students attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

The outcomes in Grade 6 Mathematics build upon students' prior learnings and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These continuing learnings prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

Outcomes describe the knowledge, skills, and understandings that students are expected to attain by the end of a particular grade level.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to know and/or be able to do in order to achieve the learnings intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can be created by teachers to meet the needs and circumstances of their students and communities.

Indicators are a representative list of the types of things a student should know or be able to do if they have attained the outcome.

Within the outcomes and indicators in this curriculum, the terms "including" and "such as", as well as the abbreviation "e.g.," occur. The use of each term serves a specific purpose. The term "including" prescribes content, contexts, or strategies that students must experience in their learning, without excluding other possibilities. For example, an indicator reads "Place a set of fractions, including whole numbers, mixed numbers, and improper fractions, on a number line and explain strategies used to determine position". This would mean that, although other types of numbers could be included at the teachers' discretion, it is mandatory that the sets of fractions must include whole numbers, mixed numbers and improper fractions.

The term "such as" provides examples of possible broad categories of content, contexts, or strategies that teachers or students may choose, without excluding other possibilities. For example, an indicator might include the phrase "such as the amount of rotation or as the angle of opening between two sides of a polygon" as examples of the types of uses for angles. This statement provides teachers and students with possible examples to consider, while not excluding others.

Finally, the abbreviation "e.g.," offers specific examples of what a term, concept, or strategy might look like. For example, an indicator might include the phrase "e.g., the construction of an octagonal drum or determining the size of a Pow Wow arena" which are specific examples that students might explore during their study of perimeter, area, and volume.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Grade 6 Mathematics outcomes are based on the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K-9 Mathematics* outcomes (2006).

Changes throughout all of the grades have been made for a number of reasons including:

- decreasing content in each grade to allow for more depth of understanding
- rearranging concepts to allow for greater depth of learning in one year and to align related mathematical concepts
- increasing the focus on numeracy (i.e., understanding numbers and their relationship to each other) beginning in Kindergarten
- introducing algebraic thinking earlier.

In Grade 6, students begin to learn about the preservation of equality within equations involving variables.

Also included in this curriculum is information regarding how Grade 6 Mathematics connects to the K-12 goals for mathematics. These goals define the purpose of mathematics education for Saskatchewan students.

In addition, teachers will find discussions of the critical characteristics of mathematics education, assessment and evaluation of student learning in mathematics, inquiry in mathematics, questioning in mathematics, and connections between Grade 6 Mathematics and other Grade 6 areas of study within this curriculum.

Finally, the Glossary provides explanations of some of the mathematical terminology used in this curriculum.

Core Curriculum

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.

(NCTM, 2000, p. 4)

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its various components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to

Core Curriculum: Principles, Time Allocations, and Credit Policy on the Ministry of Education website.

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

Developing Lifelong Learners

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

Developing lifelong learners is related to the following Goals of Education:
- *Basic Skills*
- *Lifelong Learning*
- *Self Concept Development*
- *Positive Lifestyle.*

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 Mathematics). It is crucial that the students discover the mathematics outlined in the curriculum rather than the teacher covering it.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
(NCTM, 2000, p. 20)

Developing a Sense of Self and Community

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue are exposed to a wide variety of perspectives and strategies from which to construct

Developing a sense of self and community is related to the following Goals of Education:
- *Understanding & Relating to Others*
- *Self Concept Development*
- *Positive Lifestyle*
- *Spiritual Development.*

Many of the topics and problems in a mathematics classroom can be initiated by the children themselves. In a classroom focused on working mathematically, teachers and children work together as a community of learners; they explore ideas together and share what they find. It is very different to the traditional method of mathematics teaching, which begins with a demonstration by a teacher and continues with children practicing what has been demonstrated.
(Skinner, 1999, p. 7)

Developing engaged citizens is related to the following Goals of Education:
- *Understanding & Relating to Others*
- *Positive Lifestyle*
- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change.*

a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Mathematics provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students develop their personal and social identity, and learn healthy and positive ways of interacting and working together with others.

Developing Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students such as trends in climate change, homelessness, health issues (hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students derive through mathematical analysis, they become better informed and have a greater respect for and understanding of differing opinions and possible options. With these understandings, students can make better informed and more personalized decisions regarding roles within, and contributions to, the various communities in which students are members.

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This most effectively occurs through student engagement in inquiry and problem solving when students are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "What would happen if …", "Could we find …", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

K-12 Goals for Developing Thinking:
- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

Developing Identity and Interdependence

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accept responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

K-12 Goals for Developing Identity and Interdependence:
- *understanding, valuing, and caring for oneself*
- *understanding, valuing, and caring for others*
- *understanding and valuing social and environmental interdependence and sustainability.*

Developing Literacies

Through their mathematics learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language is to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. The study of mathematics should encourage the appropriate use of technology. Moreover,

K-12 Goals for Developing Literacies:
- *developing knowledge related to various literacies*
- *exploring and interpreting the world through various literacies*
- *expressing understanding and communicating meaning using various literacies.*

students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, oral, written, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings. All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

Students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to a classmate or teacher, or when they formulate a question about something that is puzzling to them. Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols. Misconceptions can be identified and addressed. A side benefit is that it reminds students that they share responsibility with the teacher for the learning that occurs in the lesson.

(NCTM, 2000, pp. 60 – 61)

Developing Social Responsibility

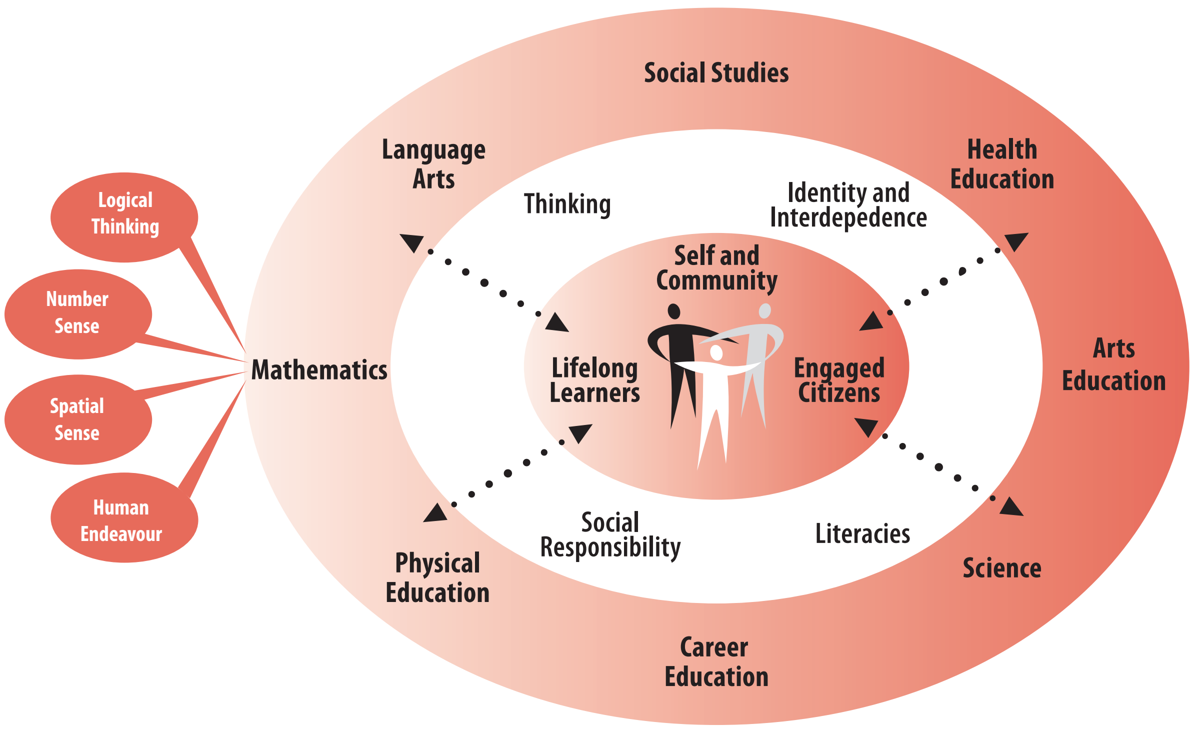
K-12 Goals for Developing Social Responsibility:

- *using moral reasoning*
- *engaging in communitarian thinking and dialogue*
- *taking social action.*

As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment be co-constructed by the teacher and students to support respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for situational validity, and strengthened.

Aim and Goals of K-12 Mathematics

The aim of the K-12 mathematics program is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities and ongoing learnings and work experiences. The K-12 mathematics program is intended to stimulate the spirt of inquiry within the context of mathematical thinking and reasoning.



Defined below are four goals for K-12 mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must, therefore, also promote student achievement with respect to the goals.

Logical Thinking

Through their learning of K-12 mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

A … feature of the social culture of [mathematics] classrooms is the recognition that the authority of reasonability and correctness lies in the logic and structure of the subject, rather than in the social status of the participants. The persuasiveness of an explanation, or the correctness of a solution depends on the mathematical sense it makes, not on the popularity of the presenter.

(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, Human, 1997, p. 10)

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observation
- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and symbolic representations)
- conjecturing and asking "what if" (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

In Grade 6, students are learning about integers for the first time. What connections and relationships should students be developing between integers and other numbers that they know?

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (including concrete, oral, physical, pictorial, and symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Students also develop understanding of place value through the strategies they invent to compute.
(NCTM, 2000, p. 82)

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

In Grade 6, students are learning to measure, compare, classify, and draw angles as well as establishing relationships between angles and triangles and quadrilaterals. What is the significance of angles in Grade 6 students' lives and in their future mathematical learnings?

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation of conjectures based upon patterns that are discovered and tested should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop.
(NCTM, 2000, p. 165)

Mathematics as a Human Endeavour

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

What types of instructional strategies support student attainment of the K-12 mathematics goals?

How can student attainment of these goals be assessed and the results be reported?

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community

- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematically and personally)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Meaning does not reside in tools; it is constructed by students as they use tools.
(Hiebert et al., 1997, p. 10)

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet their particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

The content found within the grade level outcomes for the K-12 mathematics program, and its applications, is first and foremost the vehicle through which students can achieve the four goals of K-12 mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]." This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

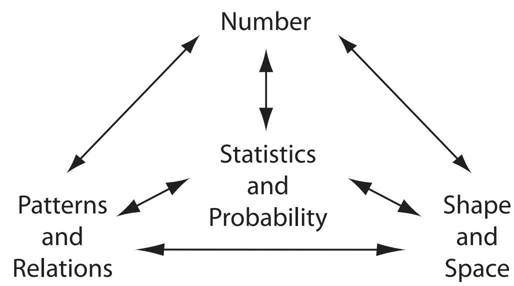
Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from covering to supporting students in discovering mathematical concepts. These facets include:

- organization of the outcomes into strands
- seven mathematical processes
- the difference between covering and discovering mathematics
- development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiqueing statements
- continuum of understanding from concrete to abstract
- modelling and making connections
- role of homework
- importance of ongoing feedback and reflection.

Strands

The content of K-12 mathematics can be organized in a variety of ways. In this curriculum, the outcomes and indicators are grouped according to four strands: **Number, Patterns and Relations, Shape and Space, and Statistics and Probability.** Although this organization implies a relatedness among the outcomes identified in each of the strands, it should be noted the mathematical concepts are interrelated across the strands as well as within strands. Teachers are encouraged to design learning activities that integrate outcomes both within a strand and across the strands so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate strands.



Mathematical Processes

This Grade 6 Mathematics curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in K-12 mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. Teachers should consider carefully in their planning those processes indicated as being important to supporting student achievement of the respective outcomes.

Communication [C]

Communication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics.
(Hiebert et al., 1997, p. 6)

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, and prior knowledge, the formal language and symbols of mathematics, and new learnings.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding for that terminology.

Concrete, pictorial, symbolic, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.
(Caine & Caine, 1991, p.5)

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

What prior and new learnings about mental mathematics and estimation will students need to apply in their study of very large and very small numbers?

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you …?", "Can you …?", or "What if …?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

Mathematical problem-solving often involves moving backwards and forwards between numerical/algebraic representations and pictorial representations of the problem.
(Haylock & Cockburn, 2003, p. 203)

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. High-order inquiry challenges students to think and develop a sense of wonder about mathematics.

In Grade 6, students should be regularly engaging in different types of mathematical reasoning during their construction and application of new understandings.

Posing conjectures and trying to justify them is an expected part of students' mathematical activity.
(NCTM, 2000, p. 191)

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Visualization [V]

[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.
(Armstrong, 1993, p.10)

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes including aspects such as dimensions and measurements.

Visualization is also important in the students' development of abstraction and abstract thinking and reasoning. Visualization provides a connection between the concrete, physical, and pictorial to the abstract symbolic. Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations as well as the use of communication to develop connections among different contexts, content, and representations.

Technology [T]

Where and how should students be using technology in their learning of Grade 6 Mathematics?

Technology tools contribute to student achievement of a wide range of mathematical outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties

- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Technology should not be used as a replacement for basic understandings and intuition.
(NCTM, 2000, p. 25)

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what must be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies, processes, rules, and problem solving, as well as the students' current and intuitive understandings of quantity and patterns. Any learning in mathematics that is a result of the logical structure of mathematics can and should be constructed by students.

What mathematical content in Grade 6 can students discover (through the careful planning of a teacher) and what does a teacher need to tell the students?

For example, in Grade 6, the students encounter percent for the first time in outcome N6.5:

> Demonstrate understanding of percent (limited to whole numbers to 100) concretely, pictorially, and symbolically.
>
> [C, CN, PS, R, V]

In this outcome, the term "percent" and the symbol "%" are both social conventions of the mathematics the students are learning and, as such, both are something that the teacher must tell the student. Representing percent and connecting it

The plus sign, for example, and the symbols for subtraction, multiplication, and division are all arbitrary convention. … Learning most of mathematics, however, relies on understanding its logical structures. The source of logical understanding is internal, requiring a child to process information, make sense of it, and figure out how to apply it.
(Burns & Silbey, 2000, p. 19)

to different types of contexts is the mathematics that students need to construct for themselves. This type of learning requires students to work concretely, physically, orally, pictorially, in writing, and symbolically. It also requires that students share their ideas with their classmates and reflect upon how the ideas and understandings of others relate to, inform, and clarify what students individually understand. In this type of learning, the teacher does not tell the students how to do the mathematics but, rather, invites the students to explore and develop an understanding of the logical structures inherent in the mathematics in increasing patterns. Thus, the teacher's role is to create inviting and rich inquiring tasks and to use questioning to effectively probe and further students' learning.

Development of Mathematical Terminology

Teachers should model appropriate conventional vocabulary.
(NCTM, 2000, p. 131)

Part of learning mathematics is learning how to speak mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that they already know or that make sense to them.

For example, in outcome SS6.3:

> Demonstrate understanding of regular and irregular polygons including:
>
> - classifying types of triangles
> - comparing side lengths
> - comparing angle measures
> - differentiating between regular and irregular polygons
> - analyzing for congruence.
>
> [C, CN, R, V]

the terminology for classifying triangles according to angles (acute, obtuse, and right) or according to side lengths (scalene, isosceles, and equilateral) will likely be unknown to the students. It is important that, before being introduced to these new words, students first explore and describe possible relationships related to angles and side lengths in triangles. Concept attainment, sorting, and defining and verifying sorting rules for pre-sorted sets are all effective teaching strategies through which students can construct an understanding of differing classifications and characteristics of triangles. Once students have developed this understanding, the new vocabulary could then be introduced.

In helping students develop their working mathematical language, it is also important for the teacher to recognize that for many students, including First Nations and Métis, they may not recognize a specific term or procedure, but the student may in fact have a deep understanding of the mathematical topic. Many perceived learning difficulties in mathematics are the result of students' cultural and personal ways of knowing not being connected to formal mathematical language.

In addition, the English language often allows for multiple interpretations of the same sentence, depending upon where the emphasis is placed. For example, consider the sentence "The shooting of the hunters was terrible" (Paulos, 1980, p. 65). Were the hunters that bad of a shot, was it terrible that the hunters got shot, was it terrible that they were shooting, or is this all about the photographs that were taken of the hunters? It is important that students be engaged in dialogue through which they explore possible meanings and interpretations of mathematical statements and problems.

First Nations and Métis Learners and Mathematics

It is important for teachers to realize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understandings. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught to their schema, cultural and environmental content, or real life experiences. A first step in actualization of mathematics from First Nations and Métis perspectives is to empower teachers to understand that mathematics is not acultural. As a result, teachers then realize that the traditional ways of teaching the mathematics are also culturally-biased. These understandings will support the teacher in developing First Nations and Métis students' personal mathematical understandings and mathematical self-confidence and ability through a more holistic and constructivist approach to learning. Teachers need to consider factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

It is important for teachers to recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others, as well as to how their own cultural background influences their current perspective and practice. Many First Nations and Métis view the world from a more holistic perspective than mathematics is often taught.

Traditionally, mathematics instruction focused on the individual parts of the whole understanding and, as a result, the contexts presented tended to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

For some First Nations and Métis students, the word "equal" may carry the cultural understanding of being "for the good of the community". For example, "equal" sharing of the meat from a hunt may not mean that everyone gets the same amount.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas between cultural groups cannot be assumed to be a direct link. Consider, for example, the concept of "equal", which is a key understanding in this curriculum. The Western understanding of "equal" is "the same". In many First Nations and Métis communities, however, "equal" is understood as meaning 'for the good of the community'. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematic classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. Constructivism, inquiry learning, and ethnomathematics allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. Ethnomathematics also shows the relationship between mathematics and cultural anthropology. It is used to translate earlier or alternative forms of thinking into modern-day understandings. Individually, and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to them personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthen the learning experiences for all.

Critiquing Statements

One way to assess students' depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. By having students critique such statements, the teacher is able to identify strengths and deficiencies in the students' understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy. For example, for outcome SP6.2, one of the indicators reads:

Critique the statement: "You can determine the sample space for an event by carrying out an experiment."

The purpose of this indicator is for teachers to assess whether students understand the difference between outcomes arising from an experiment and the theoretical outcomes. This is foundational in students developing an understanding of the role and relationships between experimental and theoretical probabilities and of probability in general.

Critiquing statements is an effective way to assess students individually or as a small or large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important that, in learning mathematics, students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a concrete starting point. Therefore, the teacher must be prepared to engage students at different points along the continuum.

It is important for students to use representations that are meaningful to them.
(NCTM, 2000, p. 140)

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. For example, when considering a problem about the total number of pencils, some students might find it more concrete to use pictures of pencils as a means of representing the situation. Other students might find coins more concrete because they directly associate money with the purchasing or having of a pencil.

As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following situational question involving subtraction:

> The elevator moved from the 3rd floor to the 8th floor and then to the 2nd level of the underground parking. How many floors did the elevator travel?

Depending upon how the question is expected to be solved (or if there is any specific expectation), this question can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures, role play), or both.

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.
(NCTM, 2000, pp. 139)

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and symbolic). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you write what you've done here using mathematical symbols?) or by having students compare their representations with others in the class.

In making these connections, students should be asked to reflect upon how the mathematical ideas and concepts that they already know are being used in their new models (e.g., I know that I can represent 1000 using the large cube of the base 10 blocks, so 10 000 could be represented by a rod of 10 of the 1000 cubes, 100 000 could be represented by a flat of 100 of the 1000 cubes, and 1 000 000 could be represented by 1000 of the 1000 cubes).

Making connections also relates to students building upon prior mathematical learnings. For example, when learning outcome P6.3:

> Extend understanding of patterns and relationships by using expressions and equations involving variables.
> [C, CN, R]

the students' prior understandings of the patterns and relationships using expressions and equations using representations involving symbols should become the starting point of their learning about the use of variables. It is not necessary for the teacher to revisit all of the ideas that students have explored in prior grades using variables rather than symbols. Rather, the meaning and role of a variable needs to be constructed from the students' current understandings of expressions and equations involving symbols.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that help students to consolidate new learnings with prior knowledge, explore possible solutions, and apply learnings to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of the drill will vary among different learners. In addition, when used as homework, drill and practice frequently serves to cause frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding of Grade 6 Mathematics, consider outcome N6.1:

> Demonstrate understanding of place value including:
>
> - greater than one million
> - less than one thousandth
> - solving situational questions using technology.
>
> [C, CN, R, PS, T]

When considering this outcome, it is important for the teacher to recognize that, prior to Grade 6, students have been studying place value both in quantities up to 1 000 000 and down to 0.999. As part of their learning, students have been exploring our decimal system for patterns and have been developing a deep understanding of the place value system. As a result, students can be asked questions such as: "How would you write a number 10 (100, 1000, 10 000 ...) times larger than 1 000 000? How would you write a number 10 (100, 1000, 10 000 ...) times smaller than 0.001? How could you concretely represent a quantity 10 times larger than 1 000 000? How would you describe how one billion compares to one million? How would each compare to one trillion? Make a prediction about your age in seconds. How could you determine it? How close was your prediction to your actual age in seconds?" These are questions that the student would take on before any classroom-based learning has been initiated regarding larger and smaller place values. The students' solutions to these types of questions help identify for the teacher the students' depth of understanding of place value and thereby inform the teacher's decisions for upcoming learning tasks within the classroom.

Characteristics of Good Math Homework

- *It is accessible to children at many levels.*
- *It is interesting both to children and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to and types of math homework offered to children throughout the year.*

(Raphel, 2000, p. 75)

Ongoing Feedback and Reflection

Feedback can take many different forms. Instead of saying, "This is what you did wrong," or "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about . . . ?"
(Stiff, 2001, p. 70)

Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.
(NCTM, 2000, p. 72)

A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences… A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.
(Haylock & Cockburn, 2003, p. 18)

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information in the teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, the addition sign (+) is something which the teacher must introduce and ensure that students know. It is the symbol used to show the combination or addition of two quantities. The process of adding, however, and the development of addition and subtraction facts should be discovered through the students' investigation of patterns, relationships, abstractions, and generalizations.

It is important for teachers to analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to consider opportunities they can provide for students to explain, apply, and transfer understanding to new situations. This analysis supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

It is important that a mathematics learning environment include effective interplay of:

- reflection
- exploration of patterns and relationships
- sharing of ideas and problems
- consideration of different perspectives
- decision making
- generalizing

- verifying and proving
- modeling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

What estimation and mental mathematics strategies might Grade 6 students be applying and/or developing as they extend their understanding of place value to very large and very small quantities?

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

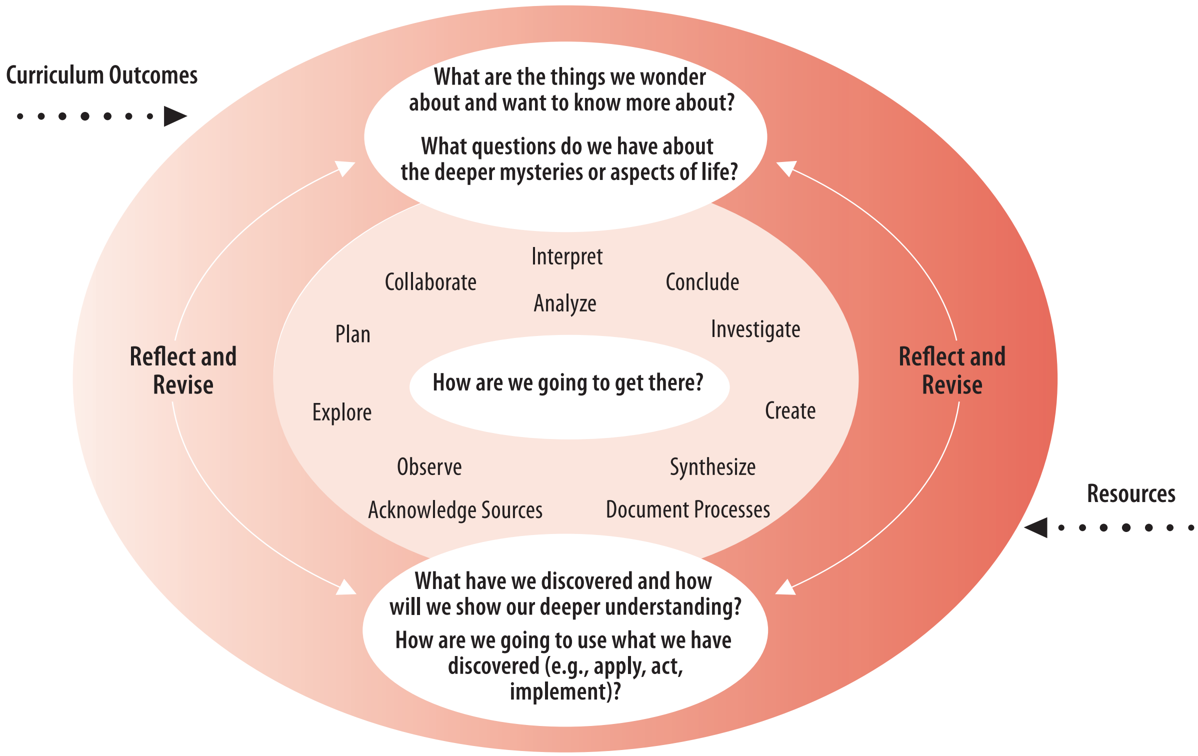
- construct deep knowledge and deep understanding rather than passively receiving it
- are directly involved and engaged in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning and mastery of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.
(Mills & Donnelly, 2001, p. xviii)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with various phases of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge. The following graphic shows the cyclical inquiry process.

Constructing Understanding Through Inquiry



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, conclude, document, reflect on learning, and develop new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students too must understand this difference.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry and give students direction for discovering deep understandings about a topic or issue under study.

Questions may be one of the most powerful technologies invented by humans. Even though they require no batteries and need not be plugged into the wall, they are tools which help us make up our minds, solve problems, and make decisions.
(Jamie McKenzie, in Schuster & Canavan Anderson, 2005, p. 1)

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions:
- *cause genuine and relevant inquiry into the important ideas and core content.*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions.*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answers.*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons.*
- *spark meaningful connections with prior learning and personal experiences.*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:
- "When would you want to add two numbers less than 100?"
- "How do you know you have an answer?"
- "Will this work with every number? Every similar situation?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning in mathematics. Questioning should also be used to encourage students to reflect on the inquiry process and the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

> - *help students make sense of the mathematics.*
> - *are open-ended, whether in answer or approach. There may be multiple answers or multiple approaches.*
> - *empower students to unravel their misconceptions.*
> - *not only require the application of facts and procedures but encourage students to make connections and generalizations.*
> - *are accessible to all students in their language and offer an entry point for all students.*
> - *lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.*
>
> *(Schuster & Canavan Anderson, 2005, p. 3)*

Reflection and Documentation of Inquiry

> *As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.*
> *(Schuster & Canavan Anderson, 2005, p. 1)*

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, drafts, models, and works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

It is important that students are required to engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

Outcomes and Indicators

Number	
Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour	
Outcomes (What students are expected to know and be able to do.)	**Indicators** (Students who have achieved this outcome should be able to:)
N6.1 Demonstrate understanding of place value including: • ***greater than one million*** • ***less than one thousandth*** ***with and without technology.*** ***[C, CN, R, PS, T]***	a. Explain, concretely, pictorially, or orally, how numbers larger than one million found in mass media and other contexts are related to one million by referencing place value and/or extending concrete or pictorial representations. b. Change the representation of numbers larger than one million given in decimal and word form to place value form (e.g., $1.8 billion would be changed to $1 800 000 000) and vice versa. c. Explain, concretely, pictorially, or orally, how numbers smaller than one thousandth found in mass media and other contexts are related to one thousandth by referencing place value and/or extending concrete or pictorial representations. d. Explain how the pattern of the place value system (e.g., the repetition of ones, tens, and hundreds), makes it possible to read and write numerals for numbers of any magnitude. e. Solve situational questions involving operations on quantities larger than one million or smaller than one thousandth (with the use of technology). f. Estimate the solution to a situational question, without the use of technology, involving operations on quantities larger than one million or smaller than one thousandth and explain the strategies used to determine the estimate.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N6.2 Demonstrate understanding of factors and multiples (concretely, pictorially, and symbolically) including: • ***determining factors and multiples of numbers less than 100*** • ***relating factors and multiples to multiplication and division*** • ***determining and relating prime and composite numbers.*** ***[C, CN, ME, PS, R]***	a. Determine the whole-numbered dimensions of all rectangular regions with a given whole-numbered area and explain how those dimensions are related to the factors of the whole number. b. Represent a set of whole-numbered multiples for a given quantity concretely, pictorially, or symbolically and explain the strategy used to create the representation. c. Explain how skip counting and multiples are related. d. Explain why 0 and 1 are neither prime nor composite. e. Analyze a whole number to determine if it is a prime number or composite and explain the reasoning. f. Determine the prime factors of a whole number and explain the strategy used to determine the factors. g. Explain how the composite factors of a whole number can be determined from the prime factors of the whole number and vice versa. h. Solve situational questions involving factors, multiples, and prime factors. i. Analyze two whole numbers for their common factors. j. Analyze two whole numbers to determine at least one multiple (greater than both whole numbers) that is common to both.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N6.3 Demonstrate understanding of the order of operations on whole numbers (excluding exponents) with and without technology. [CN, ME, PS, T]	a. Explain, with the support of examples, why there is a need to have a standardized order of operations. b. Verify, by using repeated addition and repeated subtraction for multiplication and division respectively, whether or not the simplification of an expression involving the use of the order of operations is correct. c. Verify, by using technology, whether or not the simplification of an expression involving the use of the order of operations is correct. d. Solve situational questions involving multiple operations, with and without the use of technology. e. Analyze the simplification of multiple operation expressions for errors in the application of the order of operations.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N6.4 Extend understanding of multiplication and division to decimals (1-digit whole number multipliers and 1-digit natural number divisors). [C, CN, ME, PS, R]	a. Observe and describe situations in which multiplication and division of decimals would occur. b. Explain, with justification, where the decimal place should be placed in the solution of a multiplication statement. c. Explain, with justification, where the decimal place should be placed in the solution of a division statement. d. Estimate products and quotients involving decimals. e. Develop a generalization about the impact on overall quantity when multiplied by a decimal number between 0 and 1. f. Develop a generalization about the impact on overall quantity when a decimal number is divided by a whole number. g. Solve a given situational question that involves multiplication and division of decimals, using multipliers from 0 to 9 and divisors from 1 to 9.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
N6.5 Demonstrate understanding of percent (limited to whole numbers to 100) concretely, pictorially, and symbolically. [C, CN, PS, R, V]	a. Observe and describe examples of percents (whole numbered to 100) relevant to self, family, or community, represent the percent concretely or pictorially (possibly physically), and explain what the percent tells about the context in which it is being used. b. Solve situational questions, and provide justification for possible decisions, using whole-numbered percents to 100. c. Create and explain representations (concrete, visual, or both) that establish relationships between whole number percents to 100, fractions, and decimals. d. Write the percent modeled within a concrete or pictorial representation. e. Explain why 100 is an important number when relating fractions, percents, and decimals. f. Describe a situation in which 0% or 100% might be stated.

Mathematics 6

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
N6.6 Demonstrate understanding of integers concretely, pictorially, and symbolically. [C, CN, R, V]	a. Explore and explain the representation and meaning of negative quantities in First Nations and Métis peoples, past and present. b. Observe and describe examples of integers relevant to self, family, or community and explain the meaning of those quantities within the contexts they are found. c. Compare two integers and describe their relationship symbolically using <, >, or =. d. Represent integers concretely, pictorially, or physically. e. Order a set of integers in increasing or decreasing order and explain the reasoning used. f. Identify and correct errors in the ordering of integers on a number line. g. Extend a given number line by adding numbers less than zero and explain the pattern on each side of zero. h. Explain the role of zero within integers and how it is different from other integers.

Goals: Spatial Sense, Logical Thinking, Number Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N6.7 Extend understanding of fractions to improper fractions and mixed numbers. [CN, ME, R, V]	a. Observe and describe situations relevant to self, family, or community in which quantities greater than a whole, but which are not whole numbers, occur and describe those situations using either an improper fraction or a mixed number. b. Demonstrate, concretely, pictorially, or physically, how an improper fraction and a mixed number can be used to represent the same quantity. c. Explain, with the use of concrete or visual representations, how to express an improper fraction as a mixed number (and vice versa) and write the resulting equality in symbolic form. d. Explain the meaning of a given improper fraction or mixed number by setting it into a situation. e. Place a set of fractions, including whole numbers, mixed numbers, and improper fractions, on a number line and explain strategies used to determine position. f. Respond to the question "Can quantities less than 1 be represented by a mixed number or improper fraction?".

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N6.8 Demonstrate an understanding of ratio concretely, pictorially, and symbolically. [C, CN, PS, R, V]	a. Observe situations relevant to self, family, or community which could be described using a ratio, write the ratio, and explain what the ratio means in that situation. b. Critique the statement "Ratios and fractions are the same thing". c. Create representations of and compare part/whole and part/part ratios (e.g., from a group of 3 boys and 5 girls, compare the representations boys to girls, boys to entire group, and girls to entire group – 3:5, 3:8, and 5:8 respectively). d. Express a ratio in colon and word form. e. Describe a situation in which a ratio (given in colon, word, or fractional form) might occur. f. Solve situational questions involving ratios (e.g., the ratio of students from a Grade 6 class going to a movie this weekend to those not going to a movie is 15:8. How many students are likely in the class and why?)
N6.9 Research and present how First Nations and Métis peoples, past and present, envision, represent, and use quantity in their lifestyles and worldviews.	a. Gather and document information regarding the significance and use of quantity for at least one First Nation or Métis peoples from a variety of sources such as Elders and traditional knowledge keepers. b. Compare the significance, representation, and use of quantity for different First Nations, Métis peoples, and other cultures. c. Communicate to others concretely, pictorially, orally, visually, physically, and/or in writing, what has been learned about the envisioning, representing, and use of quantity by First Nations and Métis peoples and how these understandings parallel, differ from, and enhance one's own mathematical understandings about numbers.

Patterns and Relationships

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P6.1 Extend understanding of patterns and relationships in tables of values and graphs. [C, CN, PS, R]	a. Create and describe a concrete or visual model of a table of values. b. Create a table of values to represent a concrete or visual pattern. c. Determine missing values and correct errors found within a table of values and describe the strategy used. d. Analyze the relationship between consecutive values within each of the columns in a table of values and describe the relationship orally and symbolically. e. Analyze the relationship between the two columns in a table of values and describe the relationship orally and symbolically. f. Create a table of values for a given equation. g. Analyze patterns in a table of values to solve a given situational question. h. Translate a concrete, visual, or physical pattern into a table of values and a graph (limit graphs to linear relations with discrete elements). i. Describe how a graph and a table of values are related. j. Identify errors in the matching of graphs and tables of values and explain the reasoning. k. Describe, using everyday language (orally or in writing), the relationship shown on a graph (limited to linear relations with discrete elements). l. Describe a situation that could be represented by a given graph (limited to linear relations with discrete elements). m. Research a current or past topic of interest relevant to First Nations and Métis peoples and present the data as a table of values or a graph.

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P6.2 Extend understanding of preservation of equality concretely, pictorially, physically, and symbolically. [C, CN, R]	a. Model, and explain orally, the preservation of equality for addition, subtraction, multiplication, and division concretely (e.g., balances), pictorially, or physically. b. Create, and record symbolically, equivalent forms of an equation by applying the preservation of equality (of a single operation) and verify the results concretely or pictorially (e.g., 3b = 12 is the same as 3b + 5 = 12 + 5 or 2r = 7 is the same as 3(2r) = 3(7)).
P6.3 Extend understanding of patterns and relationships by using expressions and equations involving variables. [C, CN, R]	a. Analyze patterns arising from the determination of perimeter of rectangles and generalize an equation describing a formula for the perimeter of all rectangles. b. Analyze patterns arising from the determination of area of rectangles and generalize an equation describing a formula for the area of all rectangles. c. Describe and represent geometric patterns and relationships relevant to First Nations and Métis peoples and explain how those patterns or relationships could be represented mathematically. d. Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., a + b = b + a or a × b = b × a). e. Generalize an expression that describes the relationship between the two columns in a table of values. f. Write an equation to represent a table of values. g. Generalize an expression or equation that describes the rule for a pattern (e.g., the expression 4d or the equation 2n + 1 = 8). h. Provide examples to explain the difference between an expression and an equation, both in terms of what each looks like and what each means.

Shape and Space

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS6.1 Demonstrate understanding of angles including: • ***identifying examples*** • ***classifying angles*** • ***estimating the measure*** • ***determining angle measures in degrees*** • ***drawing angles*** • ***applying angle relationships in triangles and quadrilaterals.*** ***[C, CN, ME, PS, R, V]***	a. Observe, and sort by approximate measure, a set of angles relevant to self, family, or community. b. Explore and present how First Nations and Métis peoples, past and present, measure, represent, and use angles in their lifestyles and worldviews. c. Describe and apply strategies for sketching angles including 0°, 22.5°, 30°, 45°, 60°, 90°, 180°, 270°, and 360°. d. Identify referents for angles of 45°, 90°, and 180° and use the referents to approximate the measure of other angles and to classify the angles as acute, obtuse, straight, or reflex. e. Explain the relationship between 0° and 360°. f. Describe how measuring an angle is different from measuring a length. g. Measure angles in different orientations using a protractor. h. Describe and provide examples for different uses of angles, such as the amount of rotation or as the angle of opening between two sides of a polygon. i. Generalize a relationship for the sum of the measures of the angles in any triangle. j. Generalize a relationship for the sum of the measures of the angles in any quadrilateral. k. Provide a visual, concrete, and/or oral informal proof for the sum of the measures of the angles in a quadrilateral being 360° (assuming that the sum of the measures of the angles in a triangle is 180°). l. Solve situational questions involving angles in triangles and quadrilaterals.

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS6.2 Extend and apply understanding of perimeter of polygons, area of rectangles, and volume of right rectangular prisms (concretely, pictorially, and symbolically) including: • ***relating area to volume*** • ***comparing perimeter and area*** • ***comparing area and volume*** • ***generalizing strategies and formulae*** • ***analyzing the effect of orientation*** • ***solving situational questions.*** ***[CN, PS, R, V]***	a. Generalize formulae and strategies for determining the perimeter of polygons, including rectangles and squares. b. Generalize a formula for determining the area of rectangles. c. Explain, using models, the relationship between the area of the base of a right rectangular prism and the volume of the same 3-D object. d. Generalize a rule (formula) for determining the volume of right rectangular prisms. e. Analyze the effect of orientation on the perimeter of polygons, area of rectangles, and volume of right rectangular prisms. f. Solve a situational question involving the perimeter of polygons, the area of rectangles, and/or the volume of right rectangular prisms. g. Critique the following statements using concrete or pictorial models: • "For any two right rectangular prisms, the one with the greater volume will be the prism that has the greatest base area". • "For any two rectangles, the rectangle with the greatest perimeter will also have the greatest area".

Goals: Spatial Sense, Logical Thinking, Number Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
SS6.3 Demonstrate understanding of regular and irregular polygons including: • ***classifying types of triangles*** • ***comparing side lengths*** • ***comparing angle measures*** • ***differentiating between regular and irregular polygons*** • ***analyzing for congruence.*** ***[C, CN, R, V]***	a. Observe examples of polygons, including triangles, found in situations relevant to self, family, or community and sort the polygons into irregular and regular polygons. b. Analyze the types of triangles (scalene, isosceles, equilateral, right, obtuse, and acute) to determine which, if any, represent regular polygons. c. Compare two regular polygons (using superimposing or measuring) to determine whether or not the two polygons are congruent. d. Analyze a set of regular polygons and a set of irregular polygons to identify the characteristics of regular polygons. e. Critique the following statement: "When viewed from different perspectives, the same triangle can be classified in different ways." f. Draw and classify examples of different types of triangles (scalene, isosceles, equilateral, right, obtuse, and acute) and explain the reasoning. g. Replicate a polygon in a different orientation and informally prove that the new polygon is congruent and explain the reasoning.

Mathematics 6

Goals: Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
SS6.4 Demonstrate understanding of the first quadrant of the Cartesian plane and ordered pairs with whole number coordinates. [C,CN, V]	a. Explain why the axes of the Cartesian plane should be labelled. b. Plot a point in the first quadrant of the Cartesian plane given its ordered pair. c. Analyze the coordinates of the ordered pairs of points that lie on the horizontal axis and generalize a strategy for identifying the ordered pairs of points on the horizontal axis without plotting them. d. Analyze the coordinates of the ordered pairs of points that lie on the vertical axis and generalize a strategy for identifying the ordered pairs of points on the vertical axis without plotting them. e. Explain how to plot points on the Cartesian plane given the scale to be used on the axes (by 1, 2, 5, or 10). f. Create a design in the first quadrant of the Cartesian plane, identify the coordinates of points on the design, and write or record orally directions for recreating the design. g. Generalize and apply strategies for determining the distance between pairs of points on the same horizontal or vertical line.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS6.5 Demonstrate understanding of single, and combinations of, transformations of 2-D shapes (with and without the use of technology) including: • ***identifying*** • ***describing*** • ***performing.*** ***[C, CN, R, T, V]***	a. Observe and classify different transformations found in situations relevant to self, family, or community. b. Model the translation, rotation, or reflection of 2-D shapes. c. Analyze 2-D shapes and their respective transformations to determine if the original shapes and their transformed images are congruent. d. Determine the resulting image of applying a series of transformations upon a 2-D shape. e. Describe a set of transformations, that when applied to a given 2-D shape, would result in a given image. f. Verify whether or not a given set of transformations would transform a given 2-D shape into a given image. g. Identify designs within situations relevant to self, family, or community that could be described in terms of transformations of one or more 2-D shapes. h. Analyze a given design created by transforming one or more 2-D shapes, and identify the original shape(s) and the transformations used to create the design. i. Create a design using the transformation of two or more 2-D shapes and write, or record orally, instructions that could be followed to reproduce the design. j. Describe the creation and use of single and multiple transformations in First Nations and Métis lifestyles (e.g., birch bark biting). k. Identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of the Cartesian plane). l. Perform a transformation on a given 2-D shape and identify the coordinates of the vertices of the image (limited to the first quadrant). m. Describe a transformation of a 2-D shape shown in the first quadrant of the Cartesian plane that would result in the image of the 2-D shape also being in the first quadrant.

Statistics and Probability

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP6.1 Extend understanding of data analysis to include: • ***line graphs*** • ***graphs of discrete data*** • ***data collection through questionnaires, experiments, databases, and electronic media*** • ***interpolation and extrapolation.*** ***[C, CN, PS, R, V, T]***	a. Explain the importance of accurate labelling of line graphs. b. Determine whether a set of data should be represented by a line graph (continuous data) or a series of points (discrete data) and explain why. c. Describe patterns seen in a given line graph or a graph of discrete data points, and describe a situation that the graph might represent. d. Construct a graph (line graph or a graph of discrete data points) to represent data given in a table for a particular situation. e. Interpret (through interpolation and extrapolation) the line graph or graphs of discrete data points for a situation to make decisions or solve problems. f. Observe and describe situations relevant to self, family, or community in which data might be collected through questionnaires, experiments, databases, or electronic media. g. Select a method for collecting data to answer a given question and justify the choice. h. Answer a self-generated question by performing an experiment, recording the results, graphing the data, and drawing a conclusion. i. Answer a self-generated question using databases or electronic media to collect data, then graphing and interpreting the data to draw a conclusion. j. Justify the selection of a type of graph for a set of data collected through questionnaires, experiments, databases, or electronic media.

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP6.2 Demonstrate understanding of probability by: • ***determining sample space*** • ***differentiating between experimental and theoretical probability*** • ***determining the theoretical probability*** • ***determining the experimental probability*** • ***comparing experimental and theoretical probabilities.*** ***[C, PS, R, T]***	a. Observe situations relevant to self, family, or community where probabilities are stated and/or used to make decisions. b. List the sample space (possible outcomes) for an event (such as the tossing of a coin, rolling of a die with 10 sides, spinning a spinner with five sections, random selection of a classmate for a special activity, or guessing a hidden quantity) and explain the reasoning. c. Explain what a probability of 0 for a specific outcome means by providing an example. d. Explain what a probability of 1 for a specific outcome means by providing an example. e. Explore and describe examples of the use and importance of probability in traditional and modern games of First Nations and Métis peoples. f. Predict the likelihood of a specific outcome occurring in a probability experiment by determining the theoretical probability for the outcome and explain the reasoning. g. Compare the results of a probability experiment to the expected theoretical probabilities. h. Explain how theoretical and experimental probabilities are related. i. Critique the statement: "You can determine the sample space for an event by carrying out an experiment."

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- ☑ Achievement of provincial curriculum outcomes
- ☑ Effectiveness of teaching strategies employed
- ☑ Student self-reflection on learning.

Assembling evidence from a variety of sources is more likely to yield an accurate picture.
(NCTM, 2000, p. 24)

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be in relation to curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Grade 6 Mathematics. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment should not merely be done to students; rather it should be done for students.
(NCTM, 2000, p. 22)

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

What are examples of assessments as learning that could be used in Grade 6 Mathematics and what would be the purpose of those assessments?

Assessment as learning involves student reflection on and monitoring of her/his own progress related to curricular outcomes and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

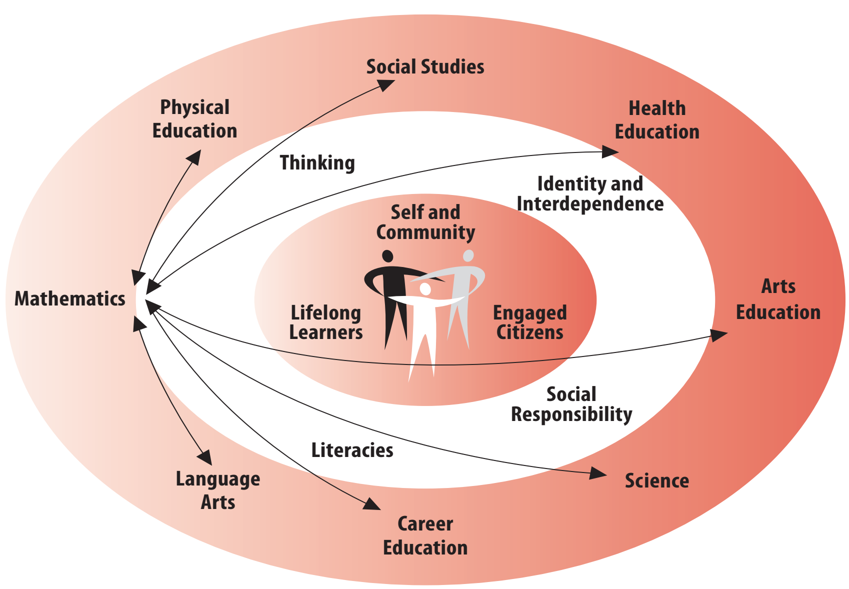
- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

Assessment should become a routine part of the ongoing classroom activity rather than an interruption.
(NCTM, 2000, p. 23)

In mathematics, students need to be regularly engaged in assessment as learning. The assessments used should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the mathematical content for Grade 6.

Connections with Other Areas of Study

There are many possibilities for connecting Grade 6 mathematical learning with the learning occurring in other subject areas. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. Following are just a few of the ways in which mathematics can be connected to other subject areas (and other subject areas connected to mathematics) at Grade 6.



Arts Education – In Grade 6 Arts Education, students explore how the arts can be used to define and represent identity. As students consider works of art, as well as create their own, they can consider the role of many of the topics in the Shape and Space strand of mathematics. Angles, polygons, and transformations are present in different art forms and the specific uses of each can help to define the identity that students are exploring. Consider, outcome SS6.1:

> Demonstrate understanding of angles including:
>
> - identifying examples
> - classifying angles
> - estimating the measure
> - determining angle measures in degrees
> - drawing angles
> - applying angle relationships in triangles and quadrilaterals.
>
> [C, CN, ME, PS, R, V]

Not only can students find examples of angles in different forms of arts, the types of angles used by the artist often communicate about the relationships and identity demonstrated within the art work. Wide or obtuse angles of movement in a dance could demonstrate anger, avoidance, or low self-esteem, while straight angles might indicate an opposite point of view or a feeling of equity. Students could analyze art works for types of angles and hypothesize the meaning that was intended to be conveyed by the artist. In addition, students could analyze the impact of different angles within works of art to help determine the types of angles that students might include in their own creations.

Career Education - In Grade 6 Career Education, students are examining how a positive self-image influences one's life. This learning can be connected to outcome SP6.1:

> SP6.1 Extend understanding of data analysis to include:
>
> - line graphs
> - graphs of discrete data
> - data collection through questionnaires, experiments, databases, and electronic media
> - interpolation and extrapolation.
>
> [C, CN, PS, R, V, T]

by having the students design an experiment to work on modifying an aspect of their self-image and to record the changes made or the impact of those changes. It is important that, in mathematics, students learn to analyze and represent both quantitative and qualitative data and a connection to Grade 6 Career Education provides opportunities for both.

English Language Arts (ELA) – ELA and Mathematics share a common interest in students developing their abilities to reflect upon and communicate about their learnings through viewing, listening, reading, representing, speaking, and writing. As an example of how mathematics involves these strands of language consider outcome N6.5:

> Demonstrate understanding of percent (limited to whole numbers to 100) concretely, pictorially, and symbolically.
>
> [C, CN, PS, R, V]

To achieve this learning outcome, students are to be explicitly engaged in understanding the meaning of percent and its relationship to quantities of which they are already aware. In developing such understandings, students first need to create and relate different representations for percents from the students' personal experiences, such as those said at home, in the media, or outside the classroom; percents within a text from another class; or percents embedded within mathematical problems students read. As the students develop their ability to read, they will also be able to engage in solving written problems involving percents. Students can share those representations with their classmates and view the representations created by others.

Through the use of language, both orally and in writing, students can communicate with their classmates and others about their developing understandings of percents and seek clarification of the ideas presented by other students. The students can be engaged in responding to what is presented to them and to questions asked of what has been presented. Such activity requires students to effectively speak, listen, and show appreciation for the ideas communicated by other students. By actively engaging in the use of language and other ways of representing their understanding, students reflect deeply upon their learning, leading to a combination of affirmations, changes, and extensions to each student's understandings of percents.

Health Education – Grade 6 Health Education provides many topics for students to explore including influences on body image and healthy decision making. These topics provide rich and engaging areas which students can investigate as part of their learning of outcome SP6.1:

> Extend understanding of data analysis to include:
>
> - line graphs
> - graphs of discrete data
> - data collection through questionnaires, experiments, databases, and electronic media
> - interpolation and extrapolation.
>
> [C, CN, PS, R, V, T]

Students can access data related to these topics through questionnaires, experiments, databases and electronic media. They can use these sources of data to seek answers to their own personal questions. The data collected can then be graphed and interpreted, and interpolations and extrapolations can be made.

Physical Education – There are many opportunities for teachers to create learning experiences that connect physical education and mathematics. These learning experiences can provide students with the opportunity to expand both their mathematical and physical skills and understandings. Outcome SP6.1:

> Extend understanding of data analysis to include:
>
> - line graphs
> - graphs of discrete data
> - data collection through questionnaires, experiments, databases, and electronic media
> - interpolation and extrapolation.
>
> [C, CN, PS, R, V, T]

provides a two-way opportunity for students to connect their physical education and mathematics learnings. First, students can collect data from carrying out their personal health-related fitness plan and their progress in improving muscular endurance, flexibility, and strength. These data can then be graphed and interpreted as part of their mathematics classroom. The students' analysis of the data can be used to assess their success in their personal health-related fitness plan and their progress in improving muscular endurance, flexibility, and strength in order to make decisions about their future actions related to these Physical Education learnings.

Science – Mathematics and science have many common concepts and processes, such as the recognition and description of patterns, sorting and categorizing, measurement, and the use of multiple representations.

In Principles of Flight, students are asked to analyze data collected from the design and production of a prototype flying object. Depending upon the performance criteria used, students may be analyzing different ratios or percents. Both ratios and percents are found within Grade 6 Mathematics (outcomes N6.5 and N6.8). The students' analysis of their prototype provides an engaging and meaningful context within which the concepts of ratios and percents can be developed and applied.

In addition, the students' study of Earth and Space Science provides opportunities for students to connect their new learnings about angles to the study of the tilt of the Earth and the impact of changing that angle. This unit of study in science also links to outcome N6.1:

> Demonstrate understanding of place value including:
>
> - greater than one million
> - less than one thousandth
>
> with and without technology.
> [C, CN, R, PS, T]

through the students' collection, study, and understanding of data related to Earth and Space which includes many quantities greater than one million and problems that involve such quantities.

Social Studies – Social studies and mathematics often connect through the investigation of patterns and trends and in the representation of data. In Grade 6, students in social studies are focusing on understanding Canada and its Atlantic neighbours. Part of the students' learning involves their investigation of cultural diversity within Canada and its Atlantic neighbours. Data which students collect as they explore this topic can be used in mathematics as a context for the students' learning of data analysis in outcome SP6.1

There is also a strong connection between social studies in Grade 6 and outcome N6.8:

> Demonstrate an understanding of ratio concretely, pictorially, and symbolically.
>
> [C, CN, PS, R, V]

Within social studies, Grade 6 students study the very specific ratio of population density. Students can create concrete and physical representations for population density ratios which will help the students visually understand what a population density is and the significance it can have. In addition, the meaningfulness of the population densities to the students' lives makes their mathematical explorations more relevant. Changing factors that influence population density will also help students more fully understand the role and meaning of ratio values.

Glossary

Angle: When two lines meet each other, angles are formed. The size of the angle is the amount of turn needed to take one line and place it on top of the other line. Angles can be classified by their measurement in degrees. Angles less than 90° are called acute angles. Angles that measure 90° are called right angles. Angles that measure between 90° and 180° are called obtuse angles. Angles that measure 180° are called straight angles.

Benchmarks: Numeric quantities used to compare and order other numeric quantities. For example, 0, 5, 10, and 20 are often used as benchmarks when placing whole numbers on a number line.

Cartesian Plane: The Cartesian plane is the result of the intersection of two perpendicular lines at a point designated as the origin. The two lines are often referred to as the x and y axes. Any point on the Cartesian plane can be represented by the pair of coordinates on the two axes that define the point. The Cartesian plane can also be called a coordinate plane.

Composite Number: A number which has factors other than 1 and itself.

Congruent: Two 2-D shapes or two 3-D objects are congruent if they have exactly the same shape and size.

Divisor: In a division statement, the quantity that is dividing into the other quantity is called the divisor.

Equality as a Balance and Inequality as Imbalance: The equal sign represents the idea of equivalence. For many students, it means "do the question". For some students, the equal sign in an expression such as 2 + 5 = means to add. When exploring equality and inequality, by using objects on a balance scale, students discover the relationships between and among the mass of the objects. The equal sign in an equation is like a scale: both sides, left and right, must be the same in order for the scale to stay in balance and the equation to be true. When the scale is imbalanced, the equation is not true. Using 2 + 5 = ☐, rather than simply 2 + 5 = helps students understand that the equal sign (=) represents equality rather than "do the work" or "do the question".

Equation: An equation is a statement which says that one expression or quantity is equal to another.

Ethnomathematics: The study of the relationship between mathematics and culture.

Experimental Probability: The probability of an event occurring that is determined by considering the results of repeated trials in an experiment.

Expression: An algebraic statement that shows a relationship between quantities and/or variables without involving equality or inequality.

Extrapolation: To use patterns and trends in values you already have for a relation in order to estimate values for the relation outside the range of what you know already.

Factor: Any whole number that will divide evenly into another whole number.

First Quadrant: One quarter of the Cartesian plane that is defined by the positive x and y axes.

Interdisciplinary: Disciplines connected by common concepts and skills embedded in disciplinary outcomes.

Integers: The set of all whole numbers plus the negative of all whole numbers.

Interpolation: Estimating a value within the known range of a relation.

Line Graphs: A graph that results from connecting consecutive ordered pairs in a relation using straight lines.

Multidisciplinary: Discipline outcomes organized around a theme and learned through the structure of the disciplines.

Multiple: A multiple of a number is exactly divisible by that number with no remainder.

Multiplier: Any quantity that another quantity is being multiplied by.

Number, Numeral, Digit: A number is the name that we give to quantities. For example, there are seven days in a week, or I have three brothers – both seven and three are numbers in these situations because they are defining a quantity. The symbolic representation of a number, such as 287, is called the numeral. If 287 is not being used to define a quantity, we call it a numeral.

Numerals, as the symbolic representation of numbers, are made up of a series of digits. The Hindu-Arabic number system that we use has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note: sometimes students are confused between these digits and their finger digits – this is because they count their fingers starting at one and get to ten rather than zero to nine.) These digits are also numerals and can be numbers (representing a quantity), but all numbers and all numerals are combinations of digits. The placement of a digit in a number or numeral affects the place value of the digit and, hence, how much of the quantity that it represents. For example, in 326, the 2 is contributing 20 to the total, while in 236 the 2 contributes 200 to the total quantity.

Object: Object is used to refer to a three-dimensional geometrical figure. To distinguish this meaning from that of shape, the word "object" is preceded by the descriptor "3-D".

Ordered Pair: A pair of numbers listed in a specific order that represents a point on the Cartesian plane. Ordered pairs are listed in brackets and the numbers are separated by a comma.

Percent: Percent means out of a hundred, being part of a hundred. The symbol for percent is %.

Personal Strategies: Personal strategies are strategies that the students have constructed and understand. Outcomes and indicators that specify the use of personal strategies convey the message that there is not a single procedure that is correct. Students should be encouraged to explore, share, and make decisions about what strategies to use in different contexts. Development of personal strategies is an indicator of the attainment of deeper understanding.

Preservation of Equality: The concept that when working with equations, equal quantities can be added, subtracted, multiplied, or divided on both sides of the equation without changing the equality of the statement.

Prime Number: A prime number is a number with exactly two factors, namely 1 and itself.

Problem: A situation or context in which a solution strategy is not immediately known, but requires being sought after.

Ratio: A comparison made between two or more quantities.

Referents: A concrete approximation of a quantity or unit of measurement. For example, seeing what 25 beans in a container looks like makes it possible to estimate the number of beans the same container will hold when it is full of the same kind of beans. Compensation must be made if the container is filled with smaller or larger beans than the referent or if the size or shape of the container is changed.

Representations: Mathematical ideas can be represented and manipulated in a variety of forms including concrete manipulatives, visual designs, sounds and speech, physical movements, and symbolic notations (such as numerals and operation signs). Students need experience in working with many different types of representations, and in transferring and translating knowledge between the different forms of representations.

Shape: In this curriculum, shape is used to refer to two-dimensional geometric figures and is thus preceded by "2-D". The term shape is sometimes also used in mathematics resources and conversations to refer to three-dimensional geometric figures. It is important that students learn to be clear in identifying whether their use of the term shape is in reference to a 2-D or 3-D geometrical figure.

Reflection: The reflection of a 2-D shape or 3-D object in a mirror line is an identical shape which has been flipped over.

Rotation: A transformation where a 2-D shape or 3-D object turns through a specific angle about a fixed point, called the centre of rotation.

Sample Space: The set of all different possible outcomes for a particular event.

Theoretical Probability: The fraction of times that a particular outcome is expected to happen.

Transdisciplinary: All knowledge interconnected and interdependent; real-life contexts emphasized and investigated through student questions.

Translation: A transformation which moves everything on a 2-D shape or 3-D object a certain amount in a certain direction.

Variable: A quantity, represented by a letter or other symbol, whose size we do not know, or whose size can sometimes change.

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Feedback Form

The Ministry of Education welcomes your response to this curriculum and invites you to complete and return this feedback form.

Document Title: **Mathematics Grade 6 Curriculum**

1. Please indicate your role in the learning community

☐ parent ☐ teacher ☐ resource teacher

☐ guidance counsellor ☐ school administrator ☐ school board trustee

☐ teacher librarian ☐ school community council member

☐ other ____

What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

☐ print

☐ online

b) Please indicate which format(s) of the curriculum you prefer:

☐ print

☐ online

3. How does this curriculum address the needs of your learning community or organization? Please explain.

4. Please respond to each of the following statements by circling the applicable number.

The curriculum content is:	Strongly Agree	Agree	Disagree	Strongly Disagree
a. appropriate for its intended purpose	1	2	3	4
b. suitable for your learning style (e.g., visuals, graphics, texts)	1	2	3	4
c. clear and well organized	1	2	3	4
d. visually appealing	1	2	3	4
e. informative	1	2	3	4

5. Explain which aspects you found to be:

   Most useful:

   Least useful:

6. Additional comments:

7. Optional:

   Name: ____________________

   School: ____________________

   Phone: ____________________ Fax: ____________________

Thank you for taking the time to provide this valuable feedback.

Please return the completed feedback form to:

Executive Director
Curriculum and E-Learning Branch
Ministry of Education
2220 College Avenue
Regina SK S4P 4V9
Fax: 306-787-2223